Helical Coil Design in DESC

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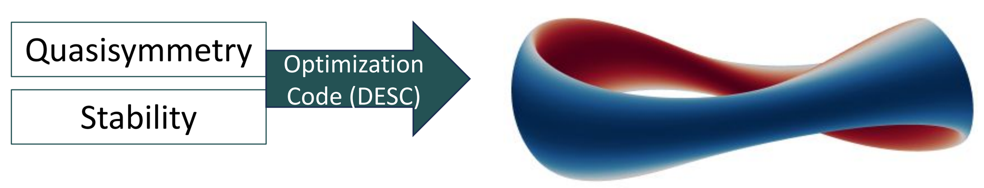
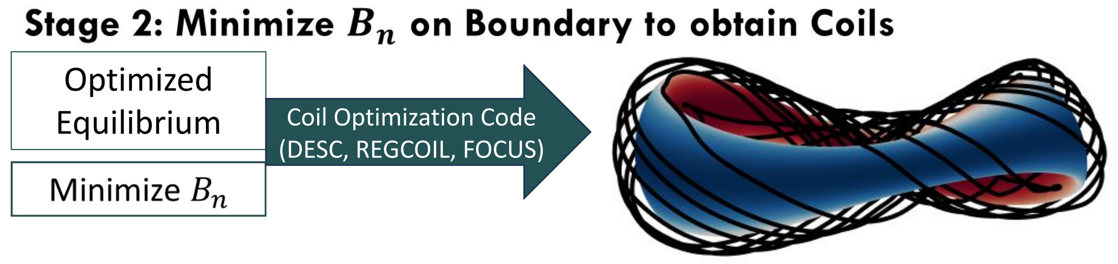
Motivation

- Stellarator optimization typically takes a two-stage approach:
  - **Stage 1: Optimize Physics Objectives To Obtain Boundary**



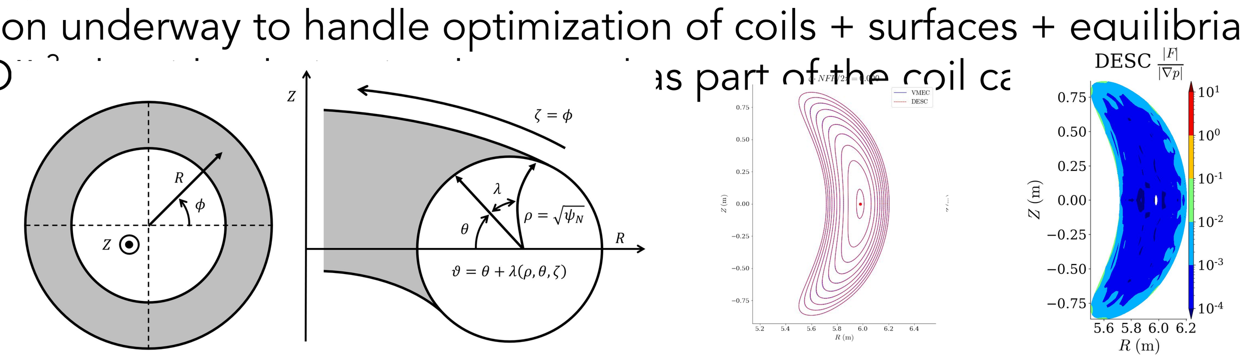
  - **Stage 2: Minimize B_n on Boundary to obtain Coils**



- **Implementing coil optimization capabilities in DESC:**
  - **Combine equilibrium + coil optimization code [cite Rogerio 1stage]**
    - Utilize DESC's automatic differentiation
  - **Helical Coil Optimization:** Limited options with current coil codes

DESC Stellarator Optimization Code

- DESC[1] is a 3D ideal MHD Stellarator Equilibrium and Optimization
- Written in Python+JAX[2] enables GPU + Automatic Differentiation capability
- Extension underway to handle optimization of coils + surfaces + equilibria
- REGCOIL ... is part of the coil ca...



REGCOIL Algorithm

- Using surface current distributions on a specified winding is an efficient approach to the coil-finding problem[4,5]

$$\boldsymbol{K} = \boldsymbol{n} \times \nabla\Phi \qquad \Phi(\theta',\zeta') = \Phi_{sv}(\theta',\zeta') + \frac{G\zeta'}{2\pi} + \frac{I\theta'}{2\pi}$$

Surface Current Density; Unit Normal to Winding Surface; Current Potential; *G*: Surface Net Poloidal Current; *I*: Surface Net Toroidal Current

- Then minimization of quadratic flux becomes a linear (in Φ_{sv}) least-squares problem, after expanding in Fourier Series (I,G, and other terms are known)

$$\chi^2_B = \int d^2a\, B^2_{\text{normal}} \qquad \Phi_{sv} = \sum_{m,n} \Phi^{mn}_{sv} \sin(m\theta' - n\zeta') \qquad B_n = B_n^{ext} + B_n^{pl} + B_n^{GI} + A\Phi^{mn}_{sv}$$

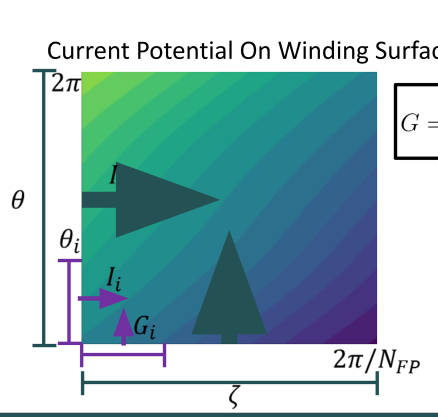
- However, can lead to poor solutions without regularization -> REGCOIL adds regularization to the problem based on the surface current

$$\chi^2_K = \int d^2a'\, K(\theta',\zeta')^2$$

- Helical coils found by specifying net toroidal current I such that $\frac{G}{I N_{FP}} \in \mathbb{Z}$
- Can calculate net toroidal or net poloidal current through a section of the surface as:

$$I = \int_0^{2\pi} \frac{\partial\Phi}{\partial\theta} d\theta = \Phi(2\pi,\zeta) - \Phi(0,\zeta)$$

$$I_i = \int_0^{\theta_i} \frac{\partial\Phi}{\partial\theta} d\theta = \Phi(\theta_i,\zeta) - \Phi(0,\zeta)$$



$$G = N_{FP} \int_0^{2\pi/N_{FP}} \frac{\partial\Phi}{\partial\zeta} d\zeta = N_{FP}(\Phi(\theta,2\pi/N_{FP}) - \Phi(\theta,0))$$

$$G_i = \int_0^{\zeta_i} \frac{\partial\Phi}{\partial\zeta} d\zeta = \Phi(\theta,\zeta_i) - \Phi(\theta,0)$$

Accounting For External Fields

- Total net poloidal current outside plasma determined by equilibrium $G^{tot} = \frac{1}{\mu_0}\int_0^{2\pi} B_\zeta d\zeta$
- External coils (TF, PF, Planar, etc) may be present outside of winding surface - must account for this!

$$G^{tot} = G + G^{ext} \qquad G^{ext} = \frac{1}{\mu_0}\int_0^{2\pi} B_\zeta^{ext} d\zeta$$

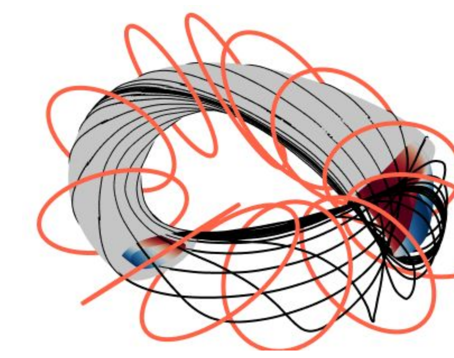
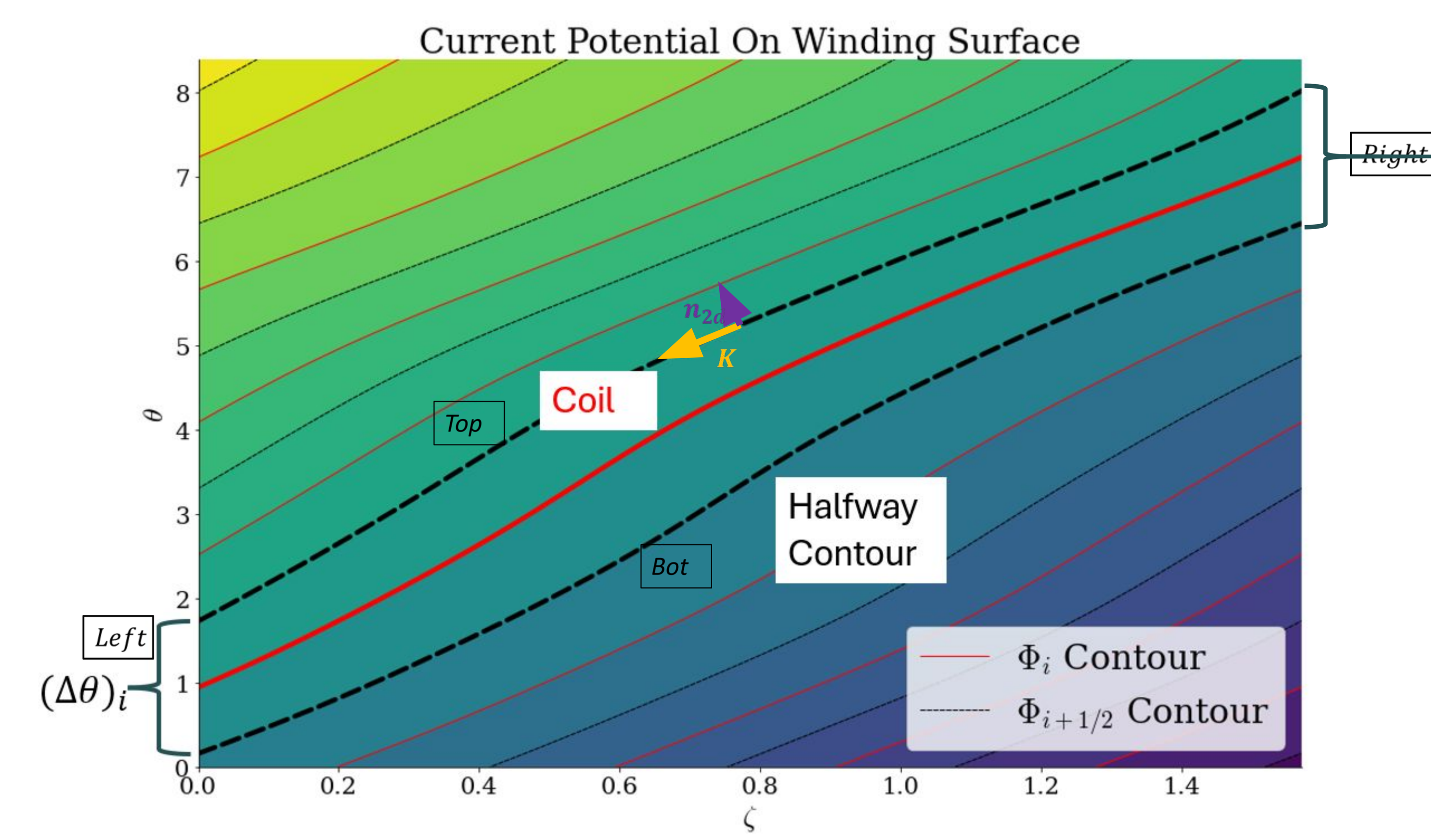


Fig: Equilibrium (red-blue) with representative winding surface (grey) with helical coils (black) inside an external planar coil set (orange)

Coil Cutting Procedure

- Each constant Φ contour is a possible coil – can choose coils by picking contours starting from evenly spaced values in θ at $\zeta = 0$



Assigning Coil Currents

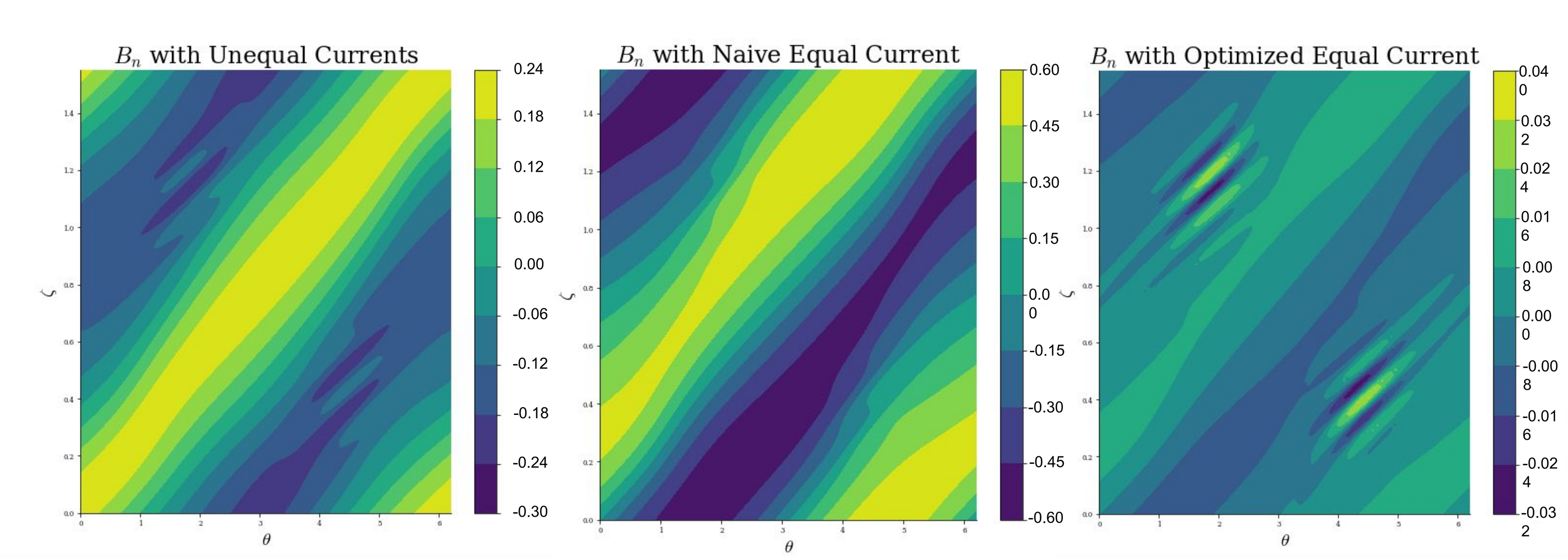
- To assign currents to the helical coils (in the absence of saddle coils), we can think of each coil has being responsible for the current up to halfway between its neighboring coils
- To find the current this represents, we only need to consider the current **entering the region from one side (due to K being divergence-free):**

$$\iint_S \nabla\cdot\boldsymbol{K} = \oint \boldsymbol{K}\cdot\boldsymbol{n_{2d}}dl = 0 = \int_{Left} \boldsymbol{K}\cdot\boldsymbol{n_{2d}}dl + \int_{Right} \boldsymbol{K}\cdot\boldsymbol{n_{2d}}dl + \int_{Top,Bot} \boldsymbol{K}\cdot\boldsymbol{n_{2d}}dl \;(=0)$$

- **Current entering left region = current exiting right region** b/c $\boldsymbol{n_{2d}} \parallel \nabla\Phi$ and $\boldsymbol{K} \perp \nabla\Phi$
- Only need to evaluate $\int_{Left} \boldsymbol{K}\cdot\boldsymbol{n_{2d}}dl = \int_{\theta_i}^{\theta_i+(\Delta\theta)_i} \frac{\partial\Phi}{\partial\theta}d\theta = \Phi(\theta_i + (\Delta\theta)_i, 0) - \Phi(\theta_i,0) =: I_i^{coil}$

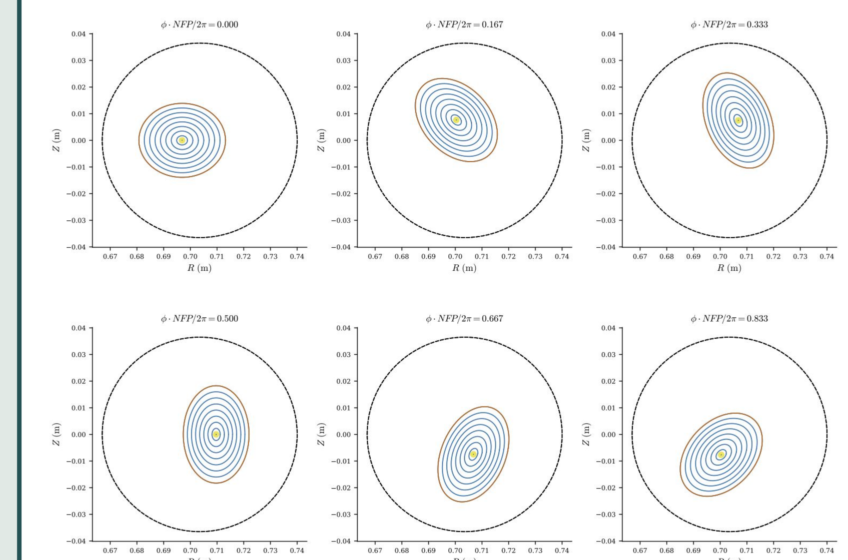
Equal Current Algorithm

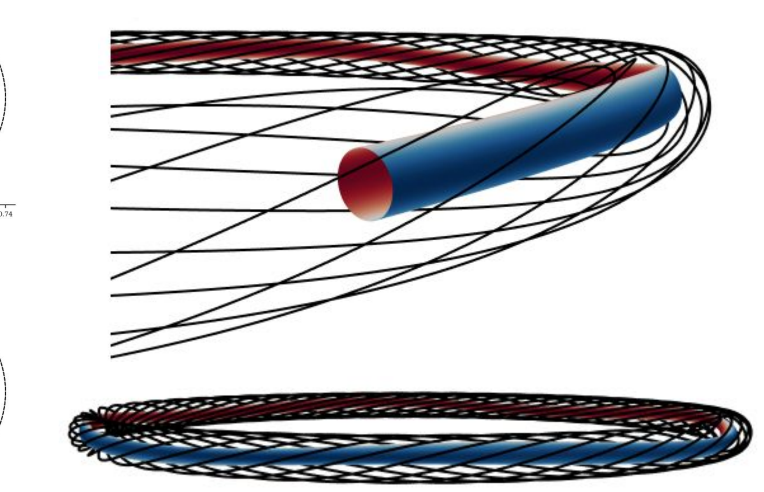
- Equal current in all coils advantageous from engineering perspective
- However, cannot naively pick equally spaced contours and set currents equal
- Instead, minimize current difference $:= f(\theta_i)$ w.r.t. θ_i $\quad f(\theta_i) = \sum (I_i^{coil}(\theta_i,\theta_{i+1}) - I/N_{coils})^2$
  - Note: Sum of currents in coils must be equal to net toroidal current I

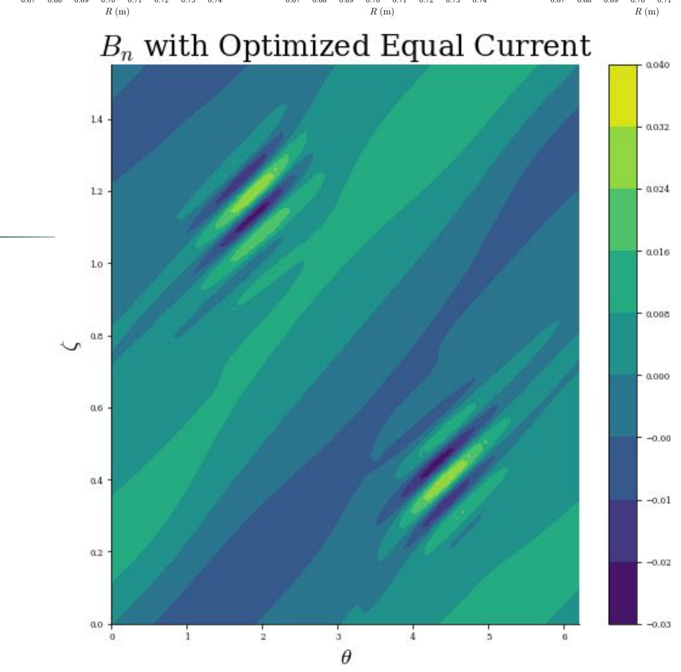


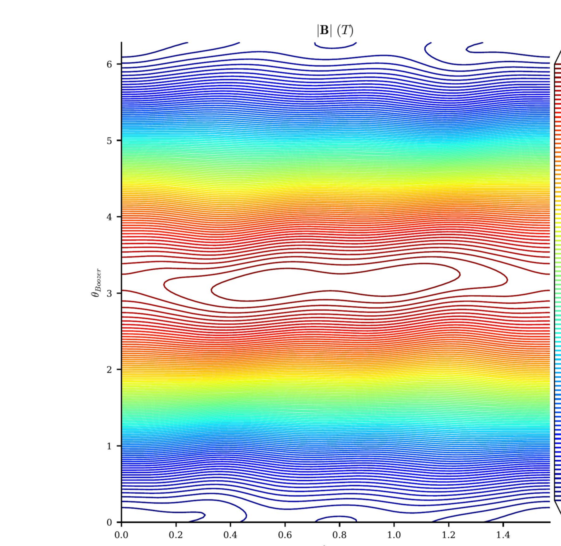
Results

- Example helical coilset for QA Equilibrium on a circular toroidal winding









- Demonstrates low normal field on surface
- Can be extended to include external solenoid

Current per Coil	73.35 KA
Number of Coils	12
Max B_{normal} on Plasma Surface	1.1e-3 T

Conclusions and Future Work

- REGCOIL algorithm implemented in Python+JAX inside of the DESC code suite
- Helical coil cutting algorithms implemented, with capability to optimize chosen coil contours so that resulting coils have equal currents
- Future work:
  - Adding filamentary coil optimization capability to DESC
  - Implement ability to handle saddle coils
  - Using REGCOIL + AD to provide coil objectives for use in stellarator optimization

References

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Acknowledgements

The authors are grateful to Matt Landreman for help with understanding the REGCOIL algorithm

This work is supported by US DoE DE-AC02-09CH11466, DESC0022005, and Field Work Proposal No. 1019

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