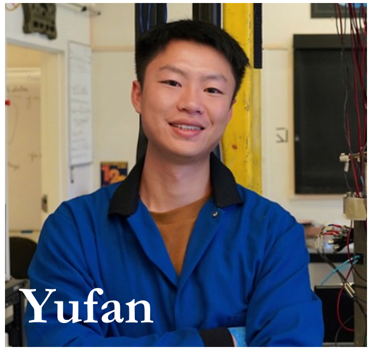
Experiments and FreeMHD Simulations of Free-Surface Liquid Metal Flows

Brian Wynne[1], Francisco Saenz[1], Jabir Al-Salami[2], Zhen Sun[3], Changhong Hu[2], Kazuaki Hanada[2], Yufan Xu[3], Egemen Kolemen[1,3]

[1]Princeton University, [2]Kyushu University, [3]PPPL



bwynne@pppl.gov

US- Japan LM Workshop. March 7th, 2024

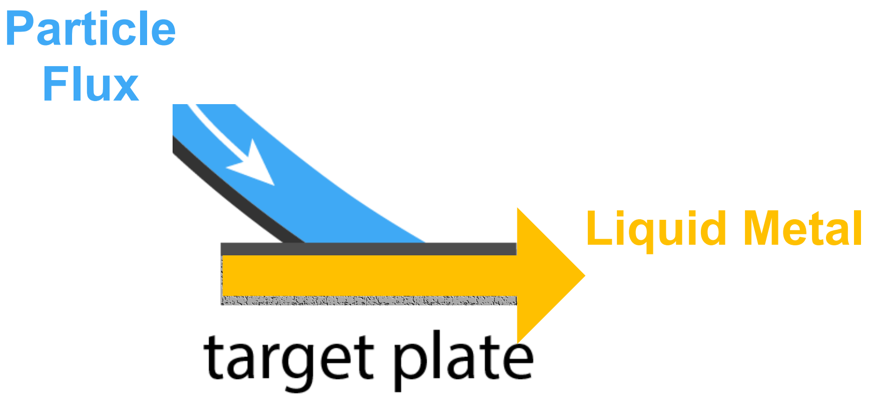
Plasma Control

Outline

- FreeMHD
- LMX-U
- JxB propulsion
- Divertorlets
- Future directions
  - LEAP

Plasma Control

Liquid Metal Flows in Open Channels



Many benefits of LM PFCs

However, issues with flowing LM:

- Severe MHD Drag
  - Flow buid up
  - High pressures needed for pumping
- Free Surface
  - Instabilities/dropject ejection
  - Waves
  - Wall detachment

Example of flow on LMX-U

Plasma Control

FreeMHD

Solver for Free Surface LM Flows

FreeMHD

- Recently developed open source MHD solver
- Free-surface liquid metal flows under strong magnetic fields
- Capable of solving fully 3D transient MHD flows

Model Details

- Electric potential formulation
- Inductionless MHD equations
- Finite-volume (FVM) OpenFOAM framework
- Volume of fluid (VoF) method

github.com/PlasmaControl/FreeMHD

Plasma Control

FreeMHD Verification

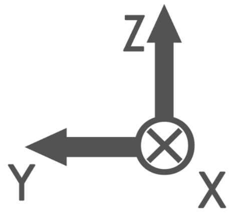
Comparison to Fully Developed Solutions

Closed Channel Setup

U, flow velocity
B, Magnetic Field across the channel
a, half width of channel
L, length of channel

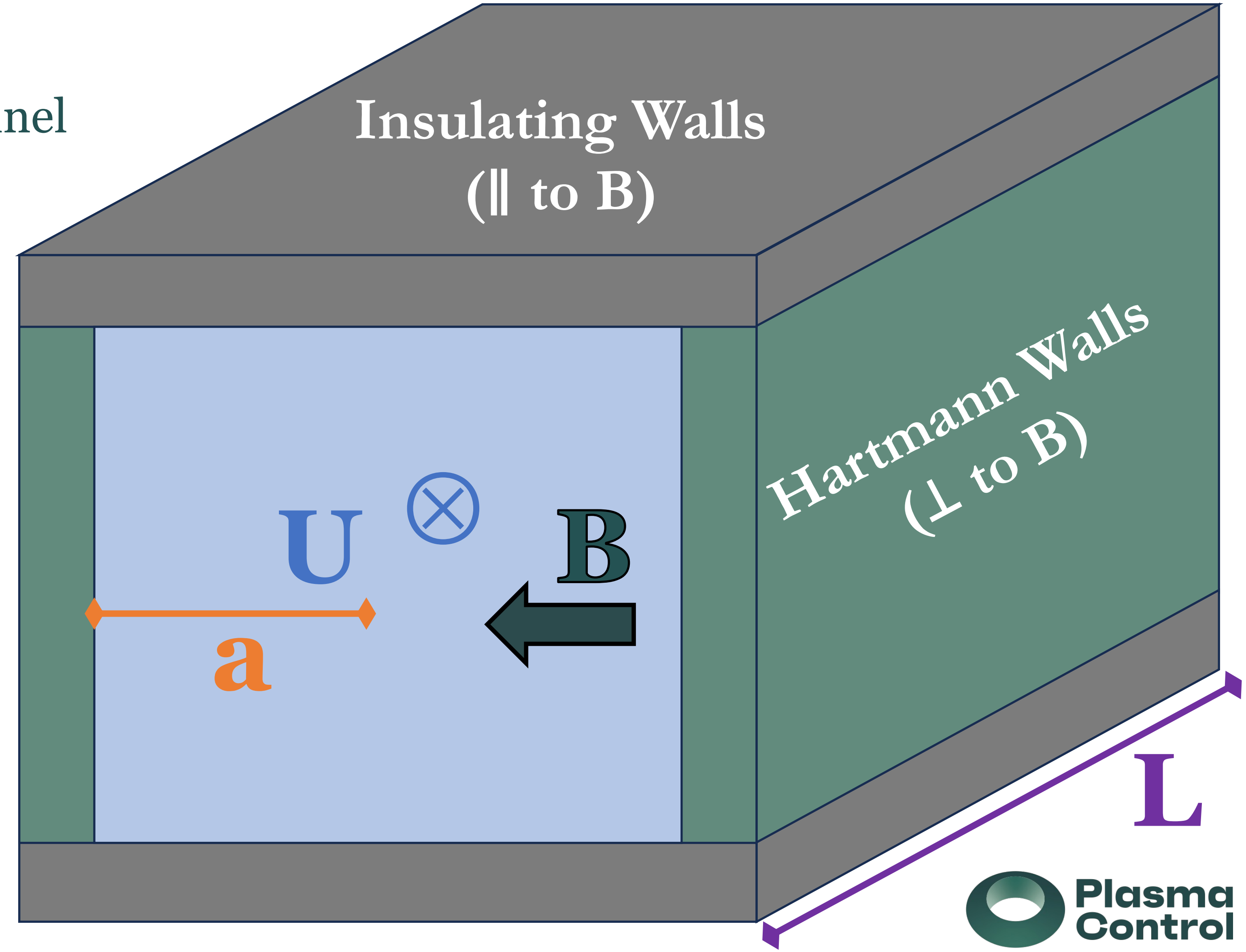
<u>Hartmann number</u>

$$\mathbf{Ha} = \mathbf{Ba}\sqrt{\frac{\sigma}{\mu}}$$

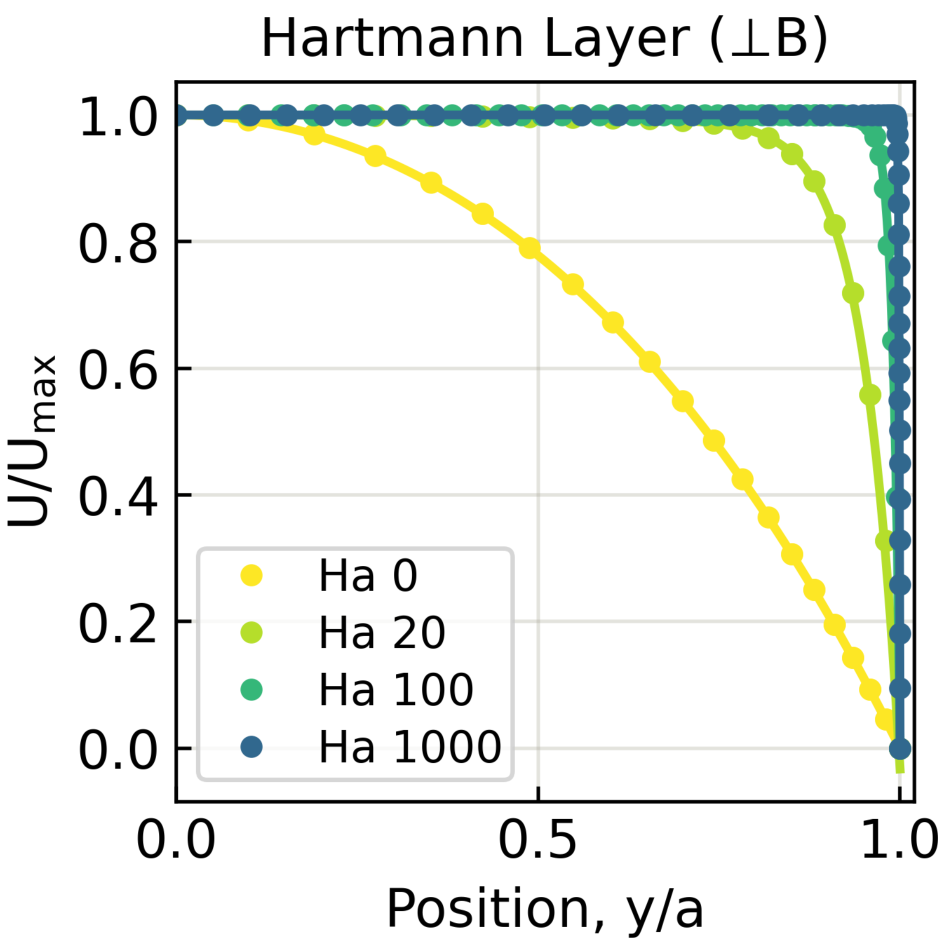


σ (electrical conductivity of fluid)
μ (viscosity of fluid)

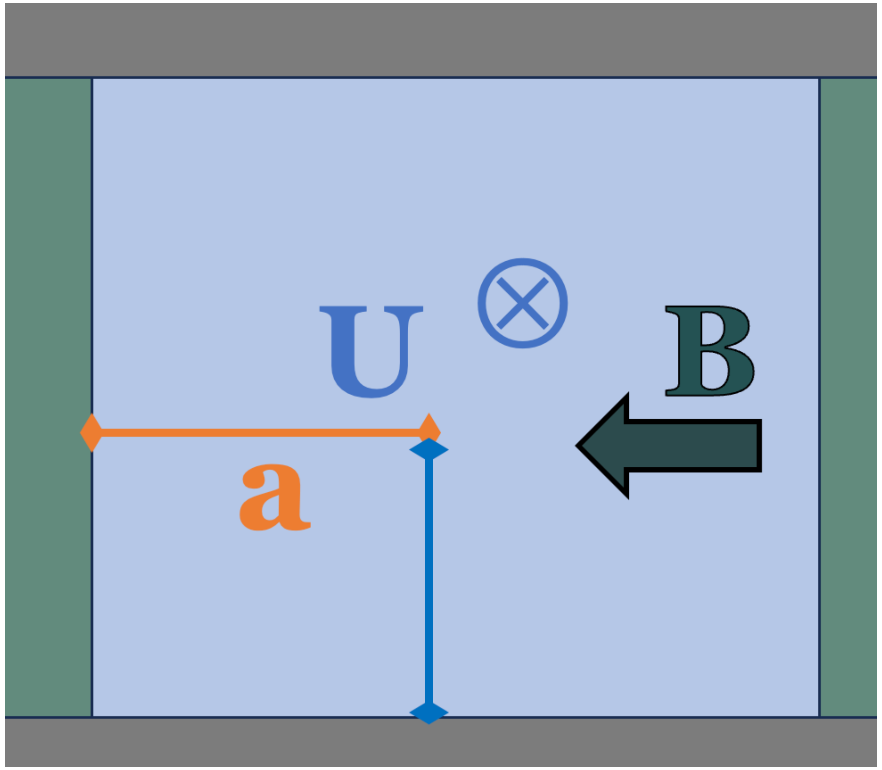
$$\mathbf{Ha}^2 \; [=] \; \frac{\textbf{Electromagnetic Forces}}{\textbf{Viscous Forces}}$$



Shercliff (Insulating Walls)



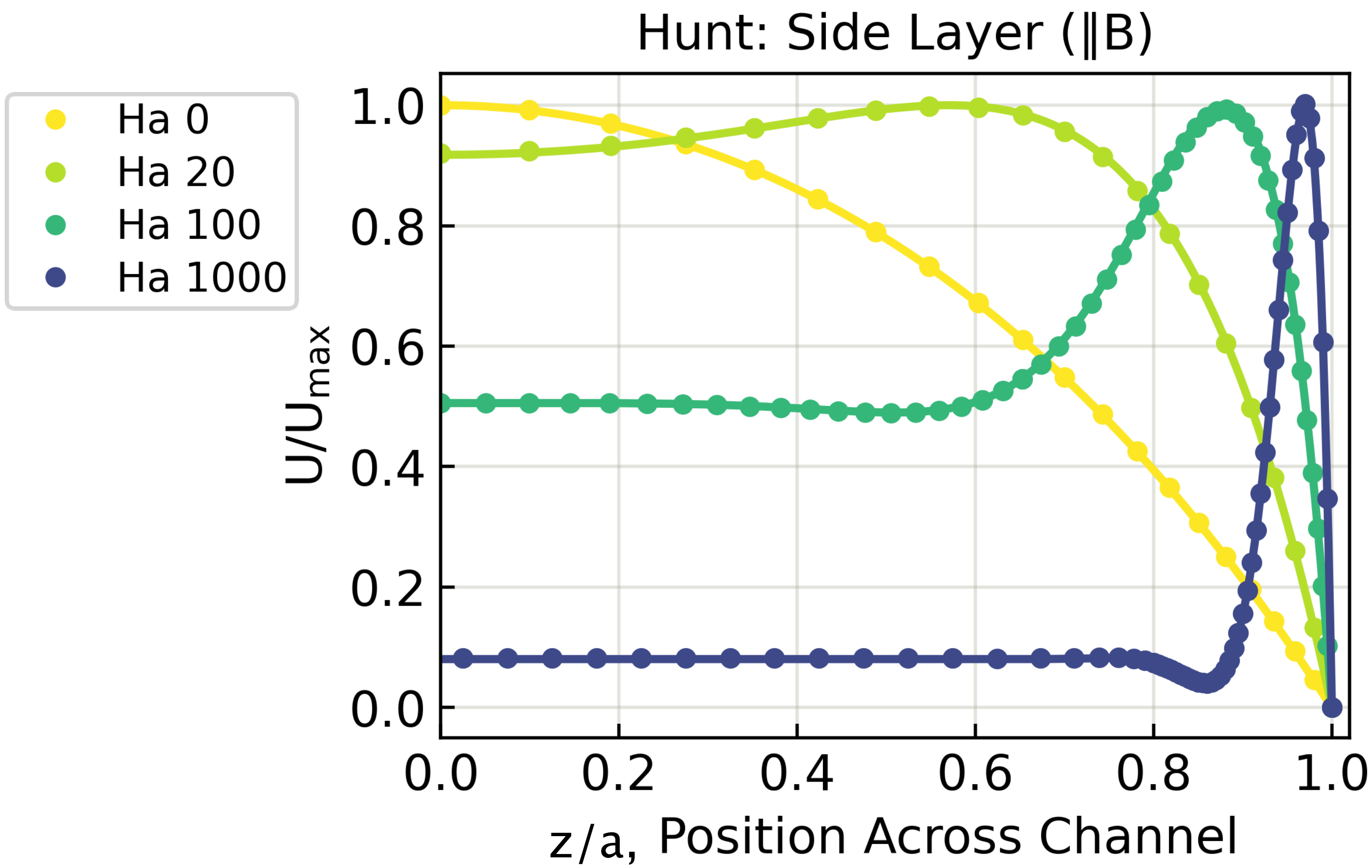
- **Lines are analytical solutions [1]**
- **Points are FreeMHD simulation**
- **Hartmann Boundary Layer scales as 1/Ha**



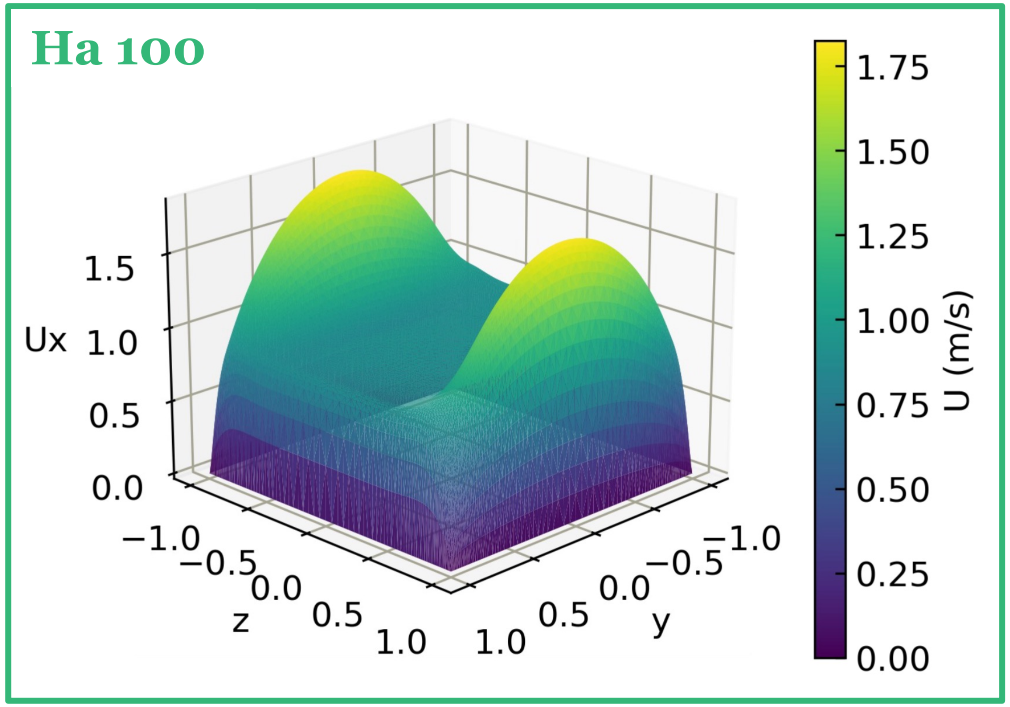
[1] JA Shercliff. "Steady motion of conducting fluids in pipes under transverse magnetic fields." In: Mathematical Proceedings of the Cambridge Philosophical Society.

Plasma Control

Hunt (Conducting Hartmann Walls)



3D Plot



- **Lines are analytical solutions [1]**
- **Points are FreeMHD simulation**
- **Hartmann Boundary Layer scales as 1/√(Ha)**

[1] JA Shercliff. "Steady motion of conducting fluids in pipes under transverse magnetic fields." In: Mathematical Proceedings of the Cambridge Philosophical Society.

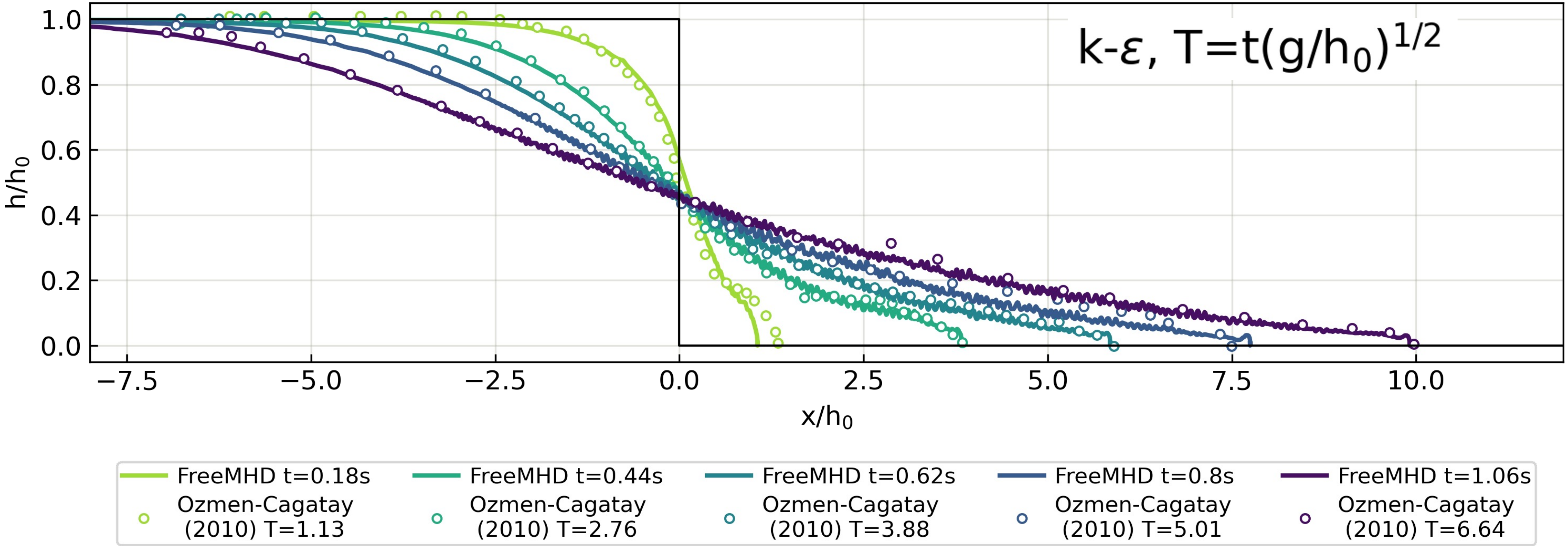
Plasma Control

FreeMHD Validation

Comparison to experiments

Dam Breaking (free surface, no magnetic field)

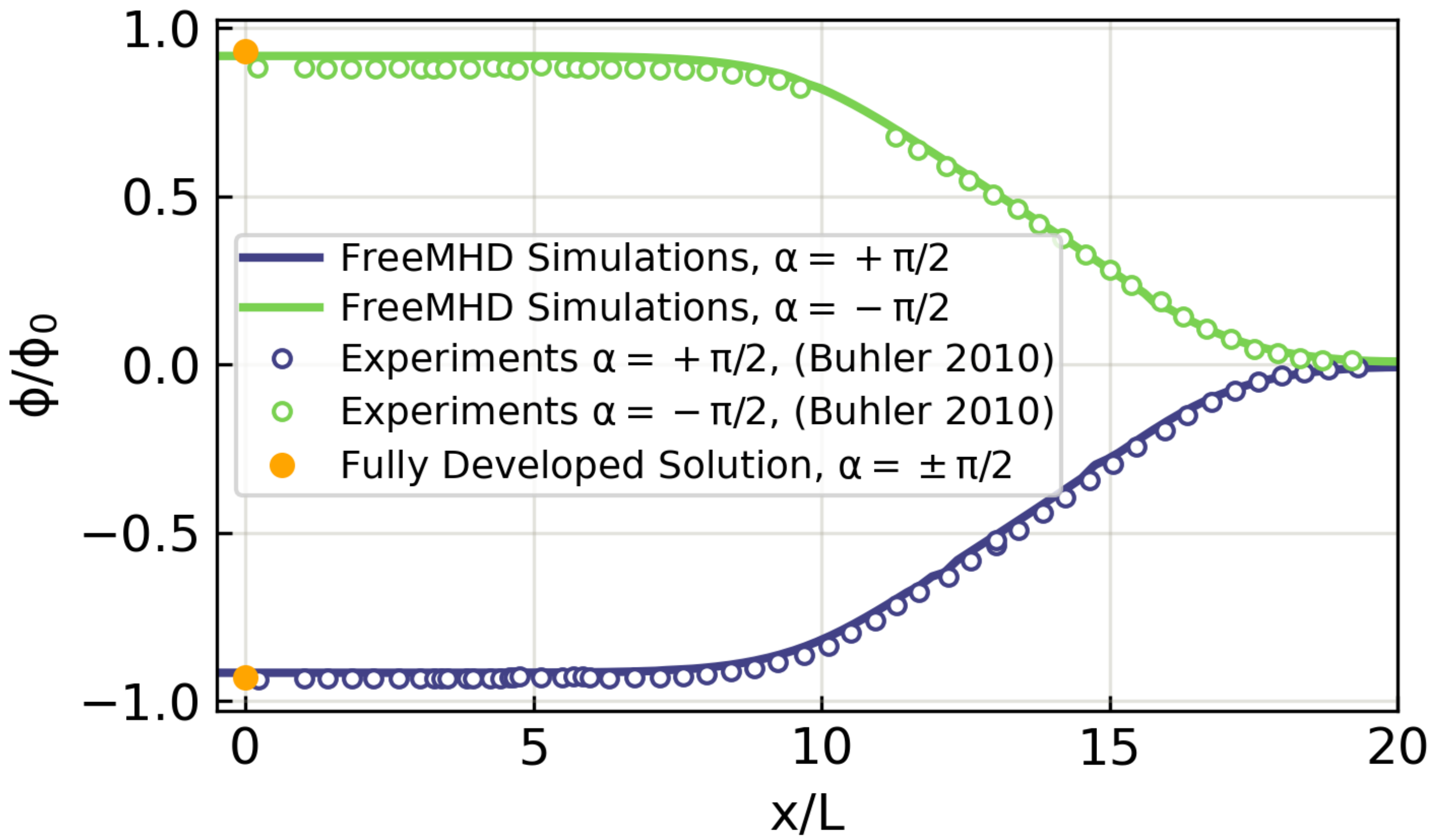
- Compare to water channel experiments (Ozmen-Cagatay, 2010)
- Validation of free surface evolution

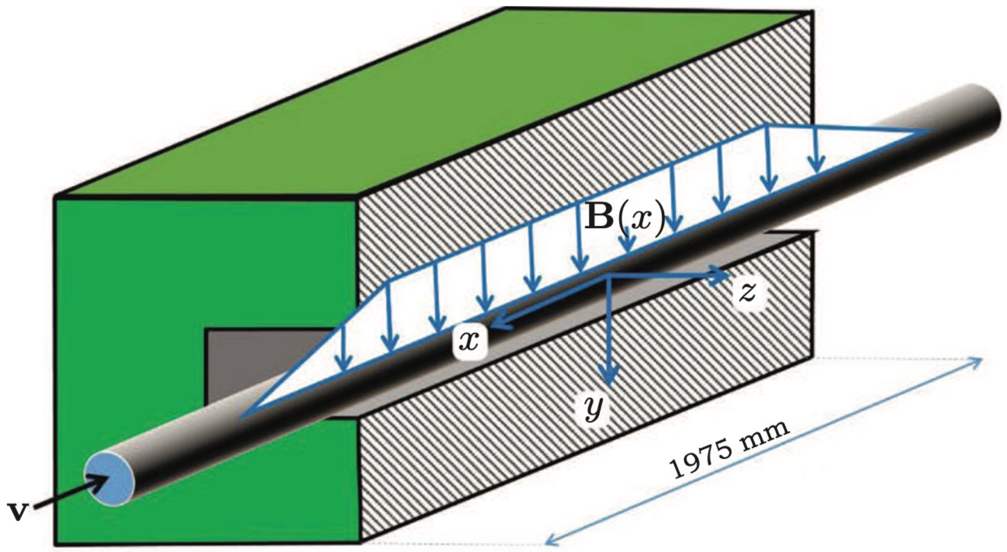


Hatice Ozmen-Cagatay & Selahattin Kocaman (2010)
Dam-break flows during initial stage using SWE and RANS approaches, Journal of Hydraulic Research, 48:5, 603-611, DOI: 10.1080/00221686.2010.507342

Fringing Magnetic Field (Closed Pipe)

- Validation of 3D current distributions
- Compare to experiments: flow into increasing B (Buhler, 2020)





Plasma Control

Bühler, L., H-J. Brinkmann, and C. Mistrangelo.
"EXPERIMENTAL INVESTIGATION OF LIQUID METAL PIPE FLOW IN A STRONG NON-UNIFORM MAGNETIC FIELD." *Magnetohydrodynamics (0024-998X)* 56 (2020).

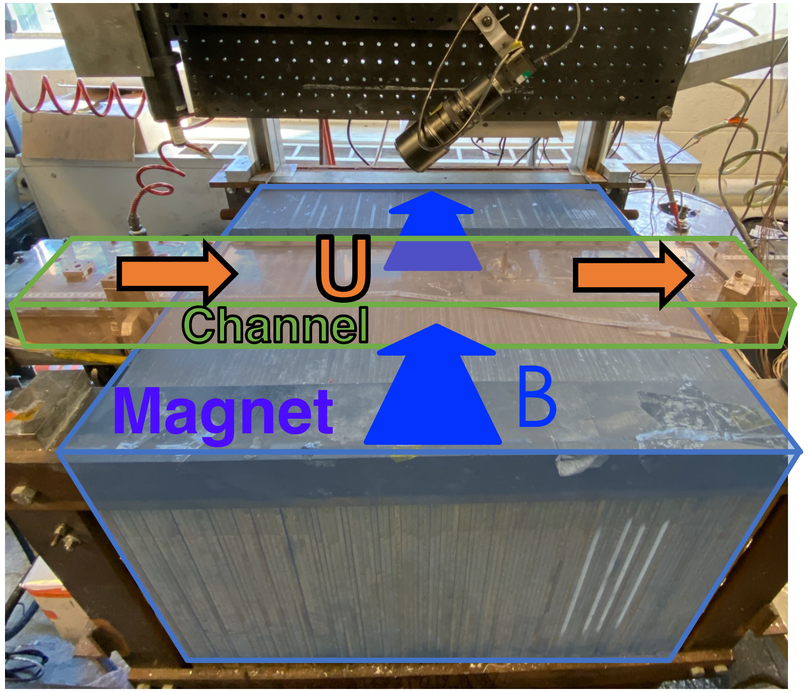
LMX-U

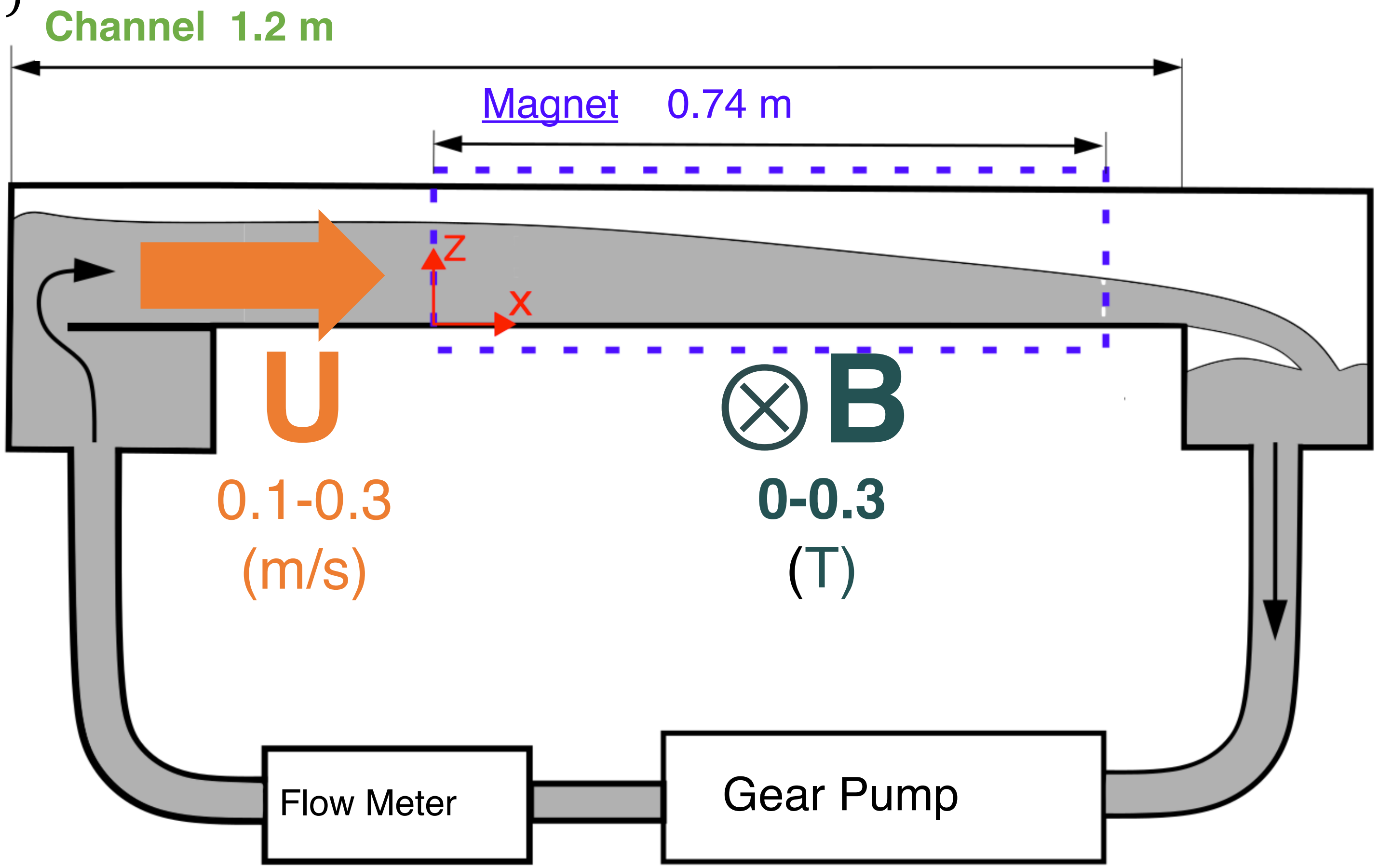
Validation of free surface LM flows

LMX-U: Methods

Liquid Metal eXperiment Upgrade (LMX-U)

- Flow loop, free surface channel
- Test bed for liquid metal experiments
- Galinstan (gallium, indium, and tin)





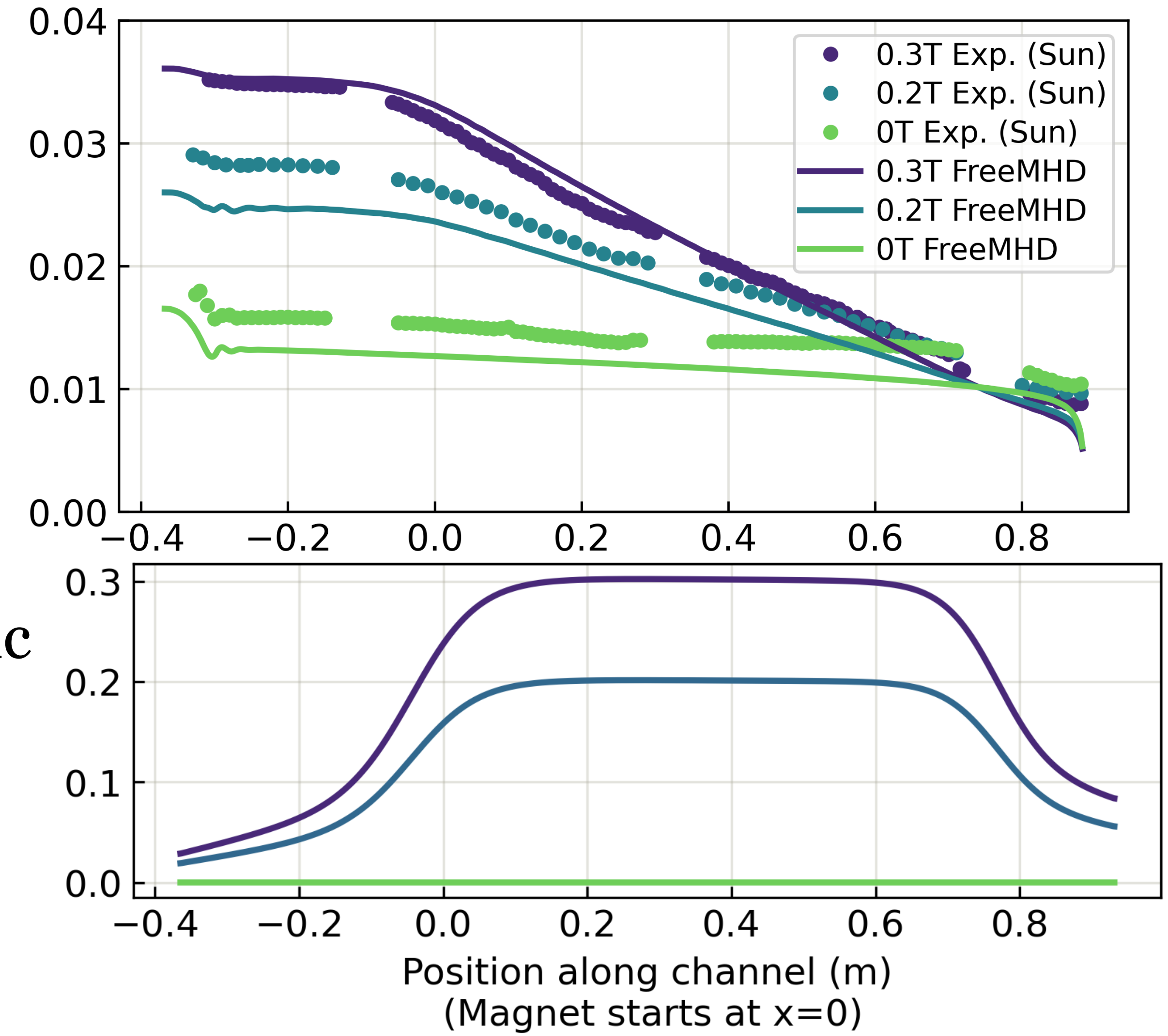
- **Trying to get stable LM flow**
- **LM flows without plasmas**
- **External magnetic field and currents**

Plasma Control

Simulations vs LMX Experiments

Liquid Height (m)

Magnetic Field (T)



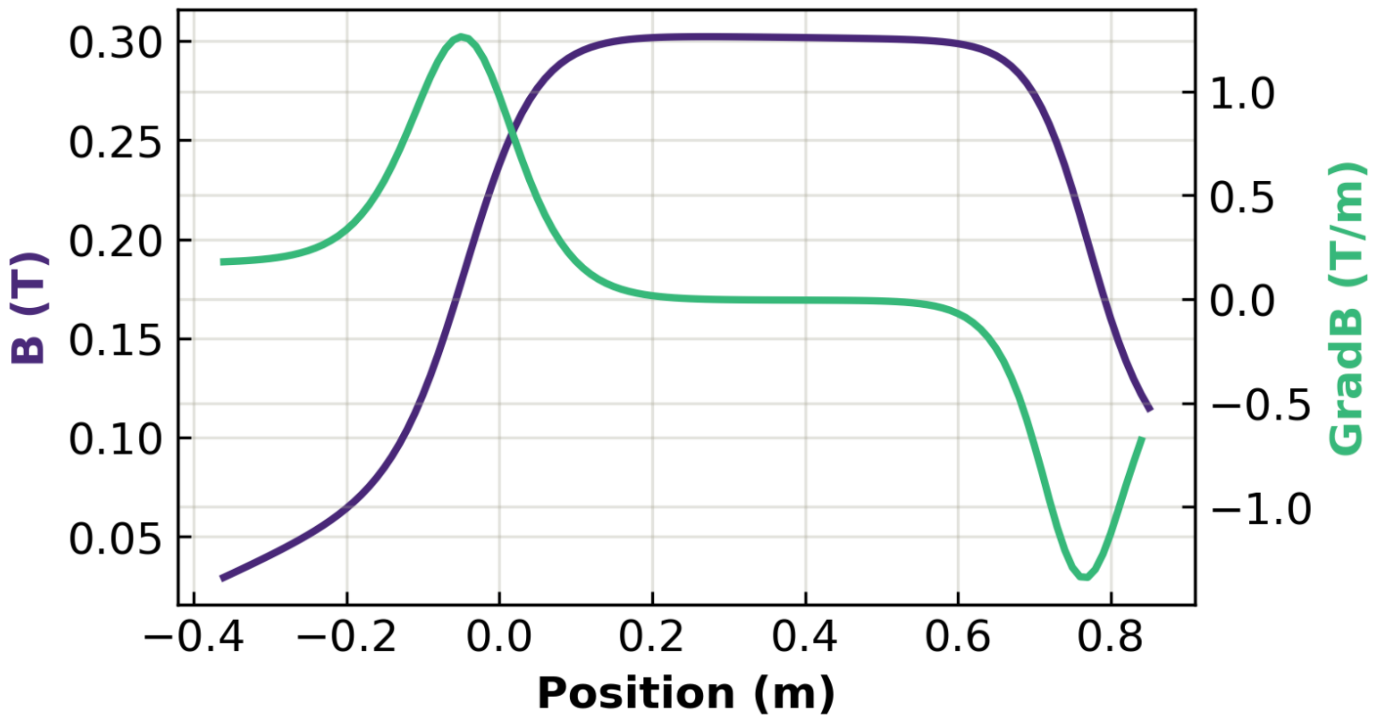
FreeMHD Simulations: 0.3T, 0.2T, 0T

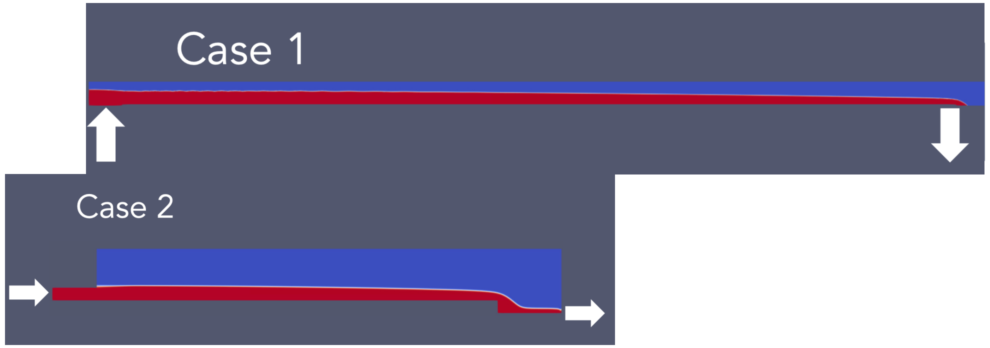
Experiments on LMX-U: 0.3T, 0.2T, 0T

- Copper 2.36mm Liner
- 0-0.3T
- Vertical inlet (No nozzle)

Plasma Control

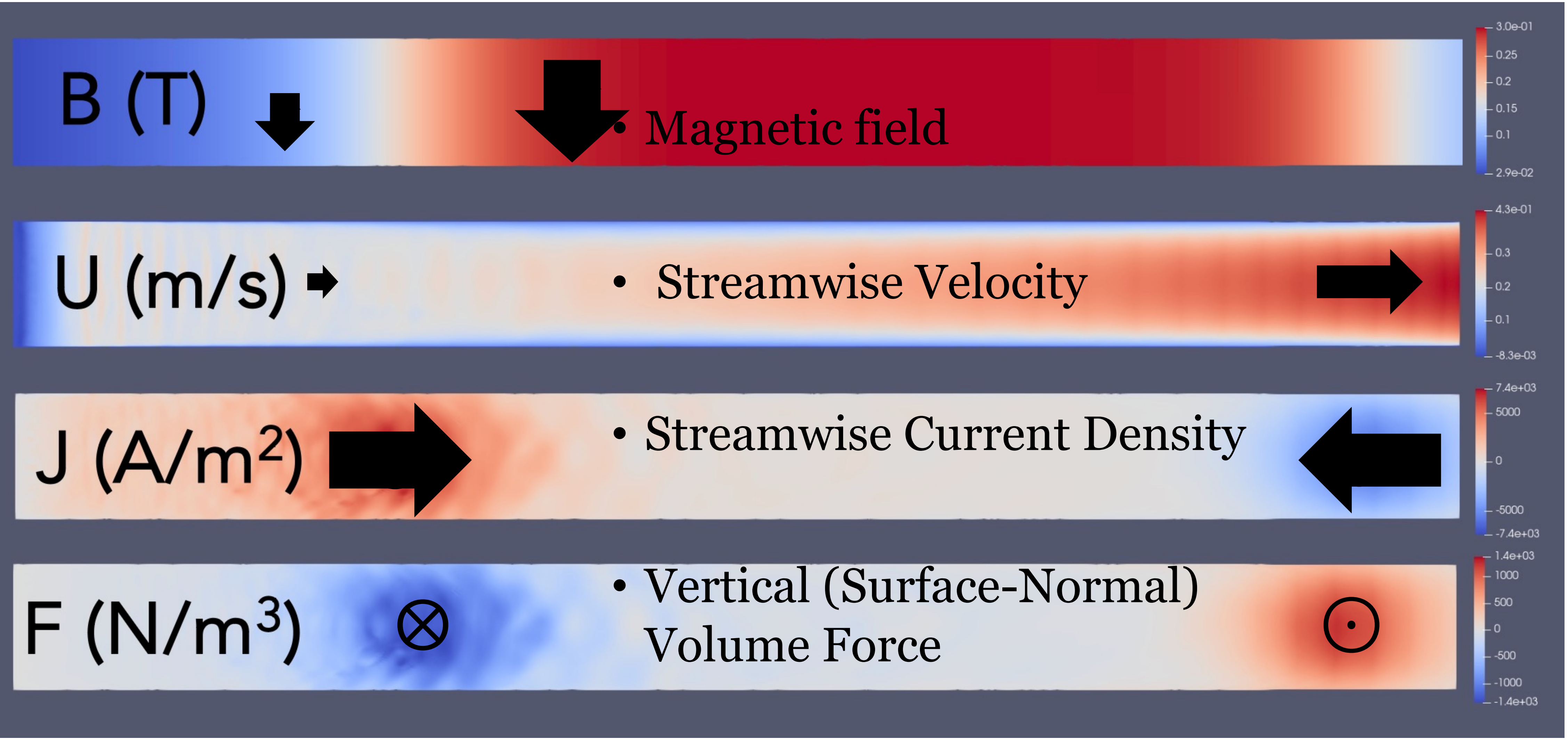
GradB in LMX





Plasma Control

GradB: Surface (FreeMHD)

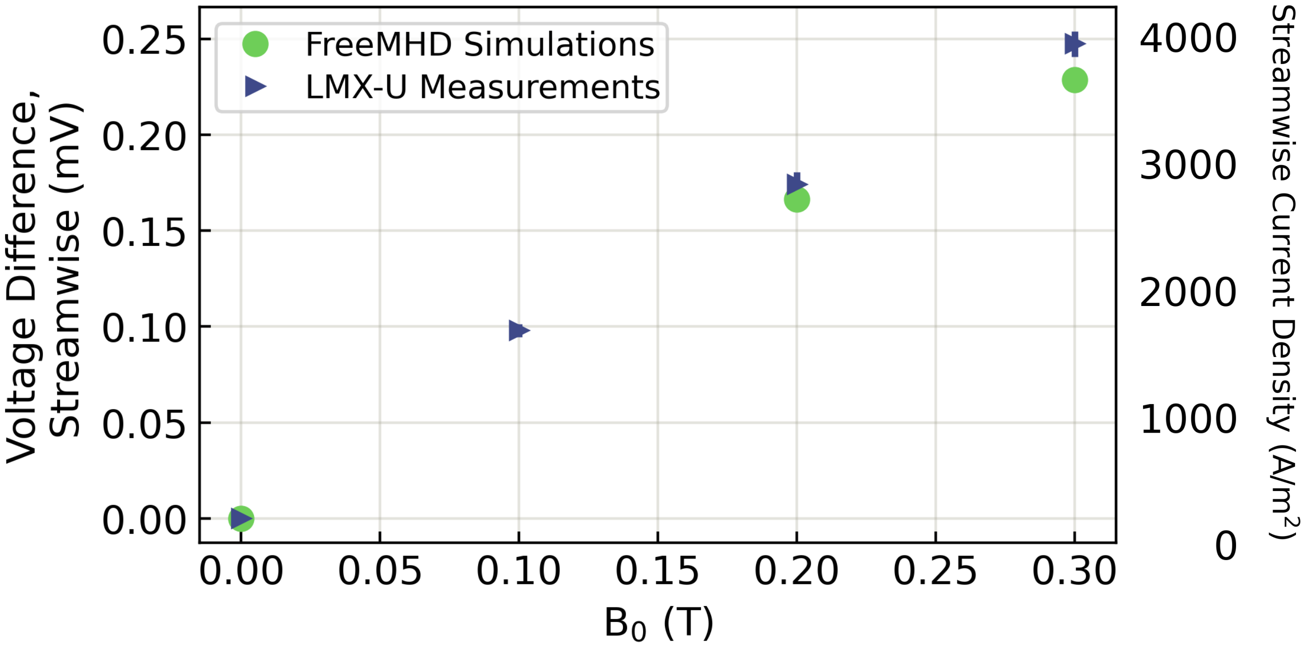


Plasma Control

GradB Free Surface Experiment/Simulations

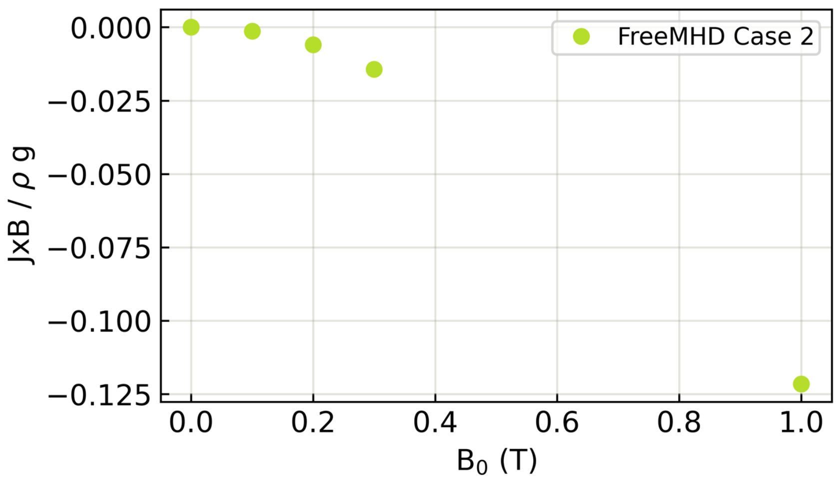
Induced Current Density

- Measured voltage difference
- Streamwise current density



Vertical Forces (Surface Normal)

- JxB force, from induced streamwise current density
  - In positive (increasing) B: Vertical force is down
  - In negative (decreasing) B: Vertical force is up
- **Main concern: streamwise currents near surface with transverse B will cause vertical forces opposing gravity**



Plasma Control

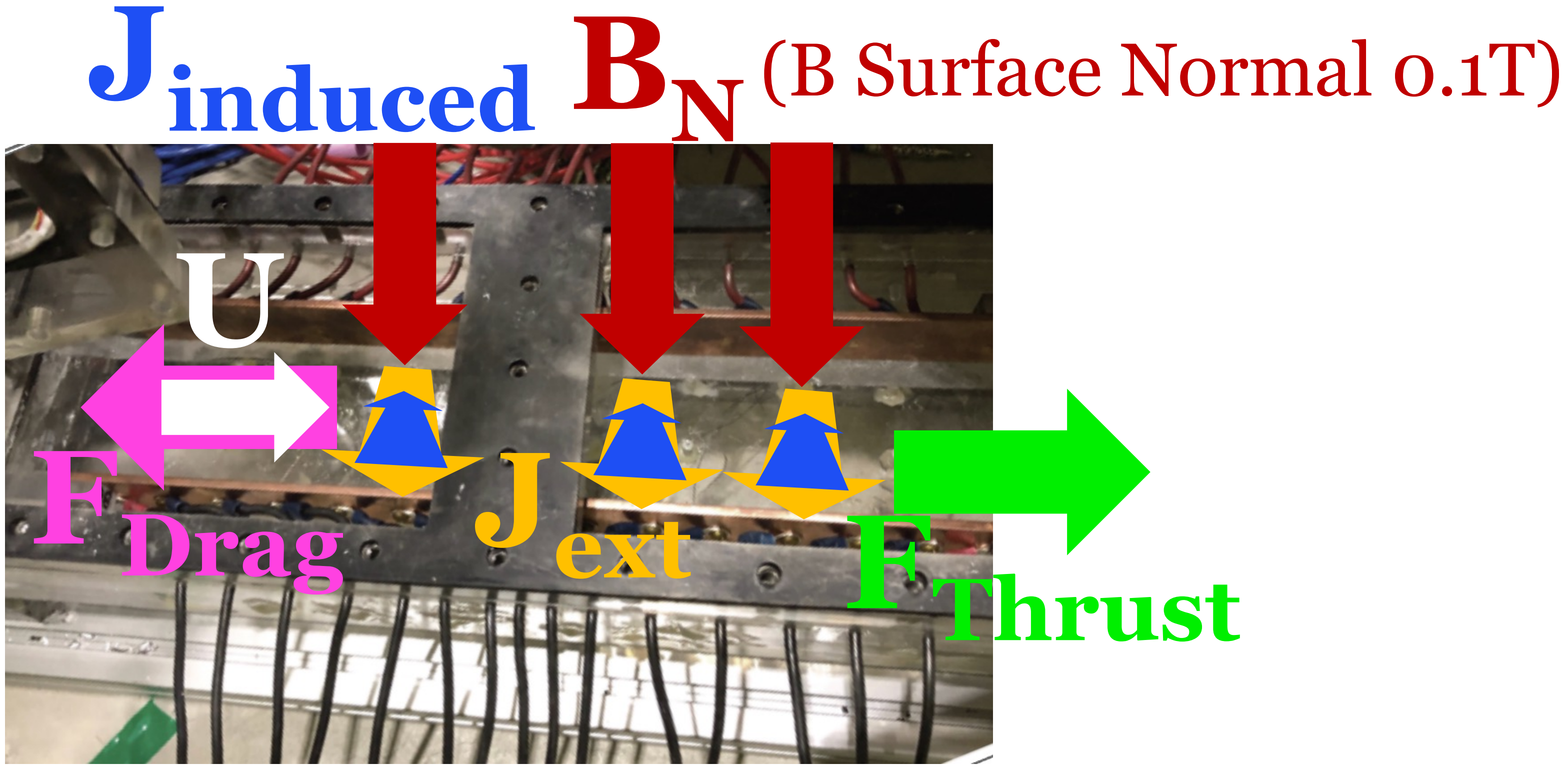
$j \times B$ propulsion

Oroshhi-2/NIFS/Kyoto University Collaboration

(F. Saenz 2023, Nucl. Fusion)

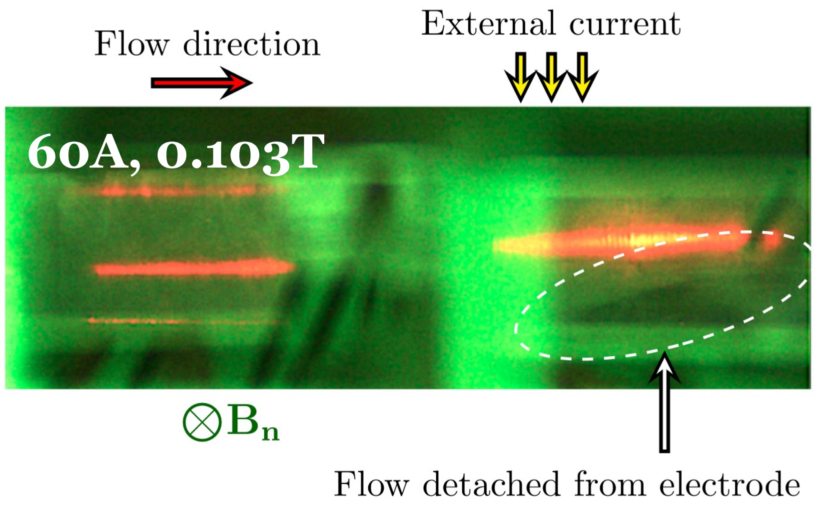
Oroshhi-2/NIFS/Kyoto University Collaboration

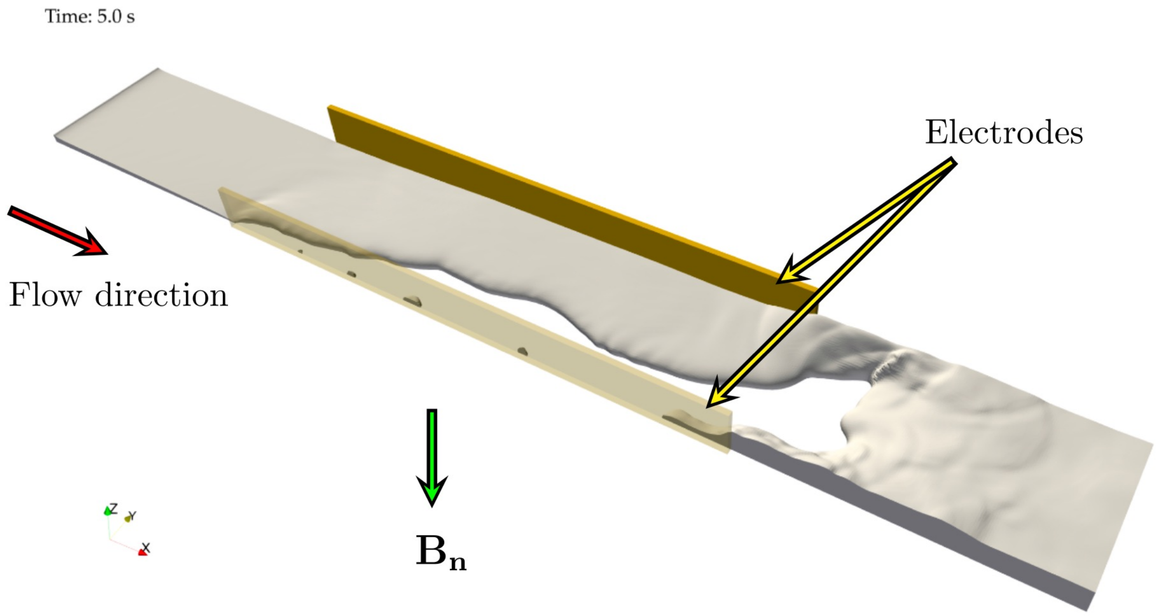
- Purpose:
  - Countering MHD drag with external currents
  - Applied currents for flow propulsion
    - To reduce effect of induced current



Experiments - Issues

- **Issue #1 Detachment from electrodes**





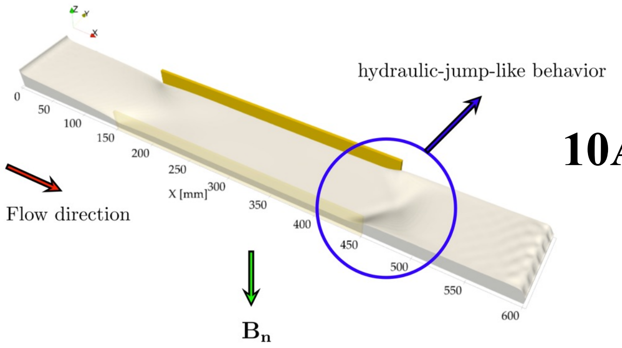
- **(Detached, but matches with simulations)**
- They countered flow detachment by increasing flow rate
- **Issue #2 → Flow buildup from downstream**
- **Takeaway: experiments were not able to demonstrate thrust due to outlet issue**

Plasma Control

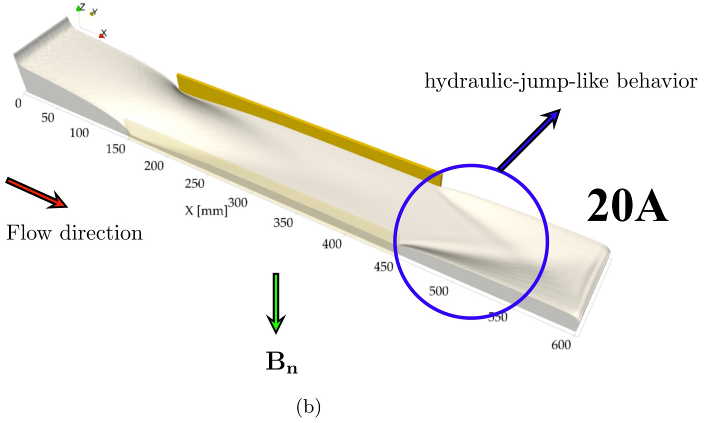
Simulation – Thrust shown

Insulated substrate

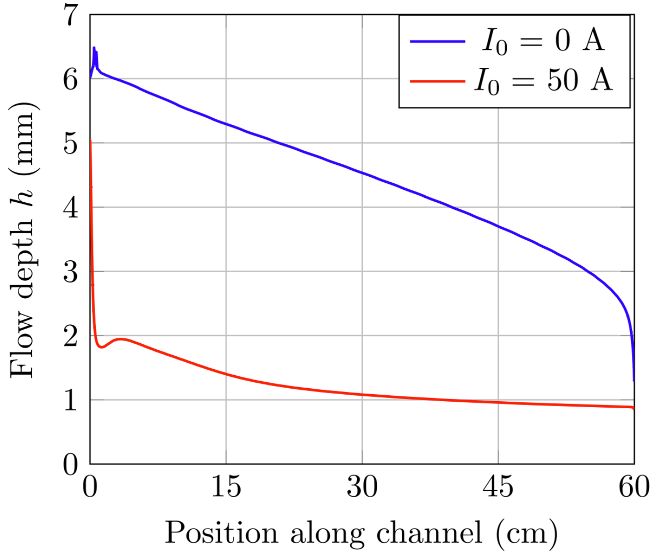
- $|B_T|$=1T and $|B_n|$=0.1T,Q=2 L/min

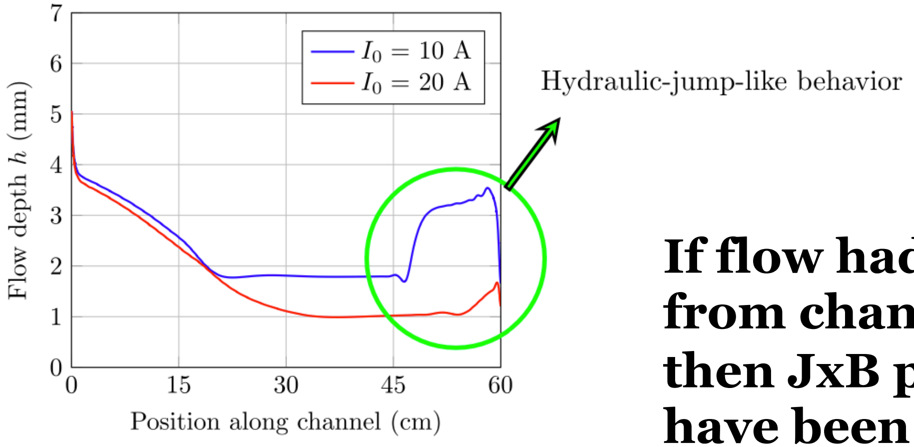


10A



20A





If flow had been removed from channel fast enough then JxB propulsion could have been demonstrated

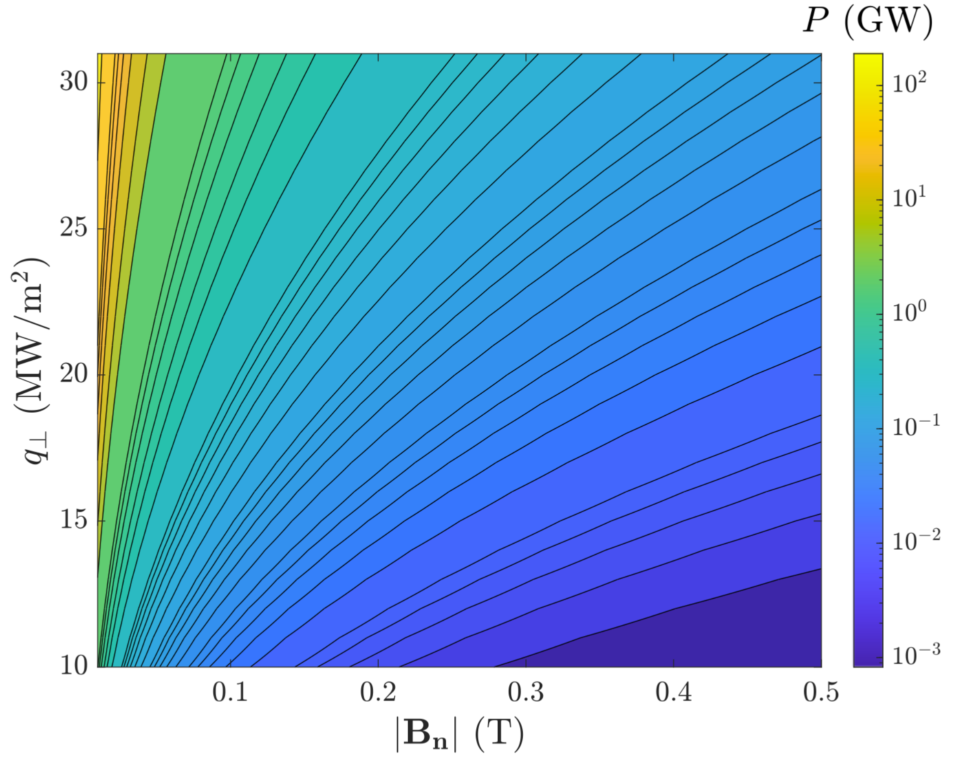
Takeaway: Shows that could be possible, but….

Plasma Control

Power requirements

The main problem:

- $\mathbf{j} \times \mathbf{B}$-thrust $\propto B_n \sim B_p$
- MHD drag $\propto B_T$



- |BT | ~ 10 T for the |BT | ~ 6 T for the outboard target. |Bn | ~ 0.1 T or less for both targets.

- Compared to the 500 MW power output expected from a fusion device like DEMO [1],
  - inner LM-divertor target could require at least 20% of this power output
  - (not even including the power requirements to pump the LM into the reactor)

- LM systems for heat exhaust in divertors should aim to require less than 5% of the total power output expected from a fusion device [2]

[1] EUROFusion 2022 The demonstration power plant: DEMO (available at: www.euro-fusion.org/programme/demo/)

[2] Fisher A., Sun Z. and Kolemen E. 2020 Liquid metal "divertorlets" concept for fusion reactors Nucl. Mater. Energy 25 100855

Plasma Control

JxB Propulsion

- Overall, difficult to achieve stable configuration, and **even if JxB Propulsion using B_p is successful, power requriments will be too extreme**
- Instead one should take advantage of externally injected currents that generate a source of thrust that is proportional the toroidal magnetic field B_T
- **(Divertorlets concepts has a 'pumping force' that is proportional to the toroidal magnetic field, which allows them to operate with small power requirements)**

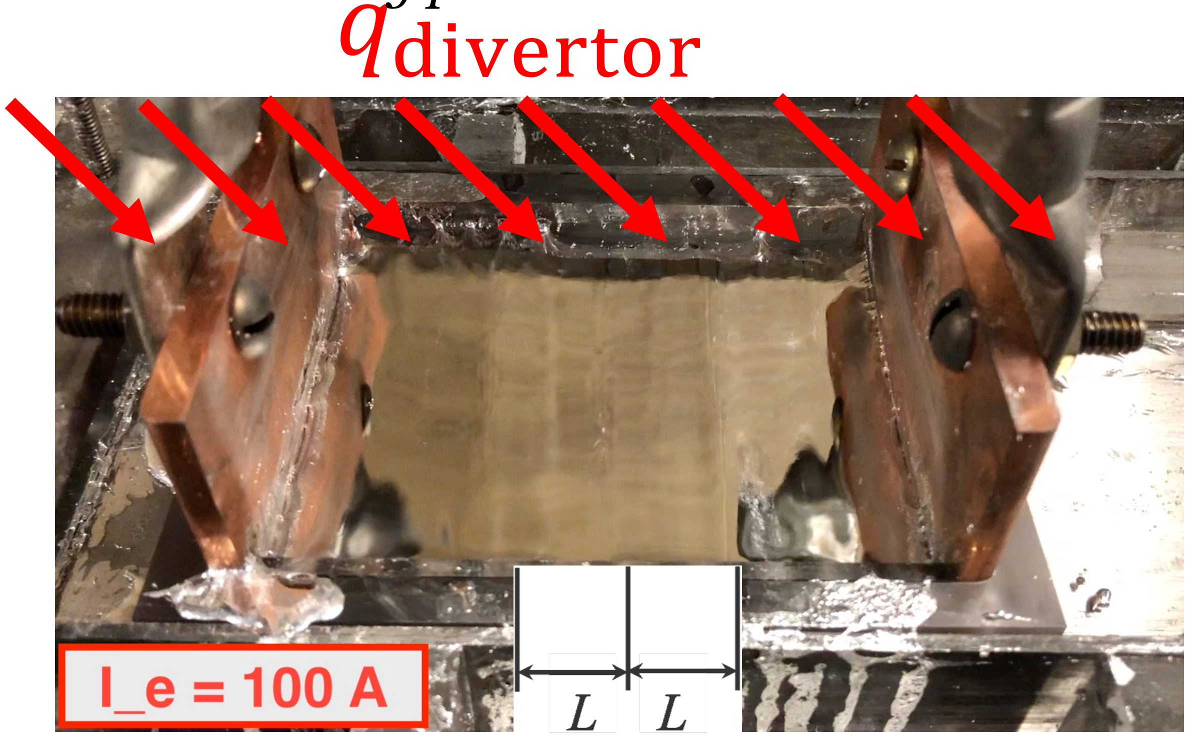
Plasma Control

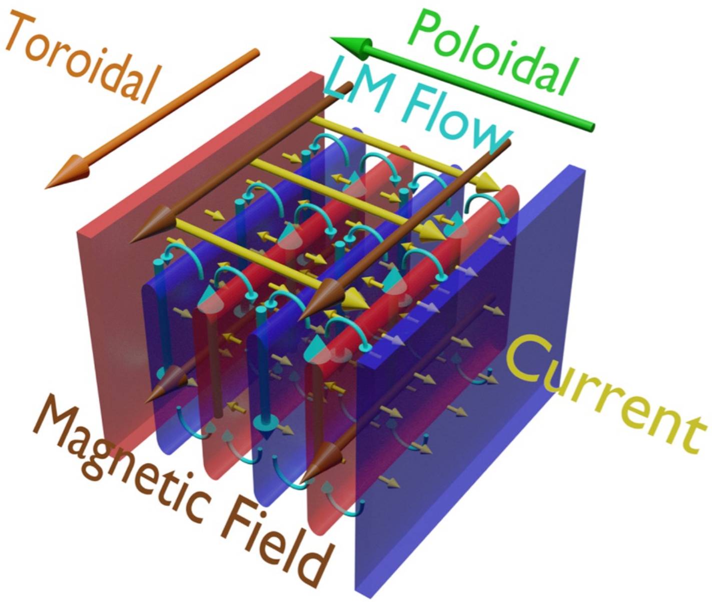
Divertorlets

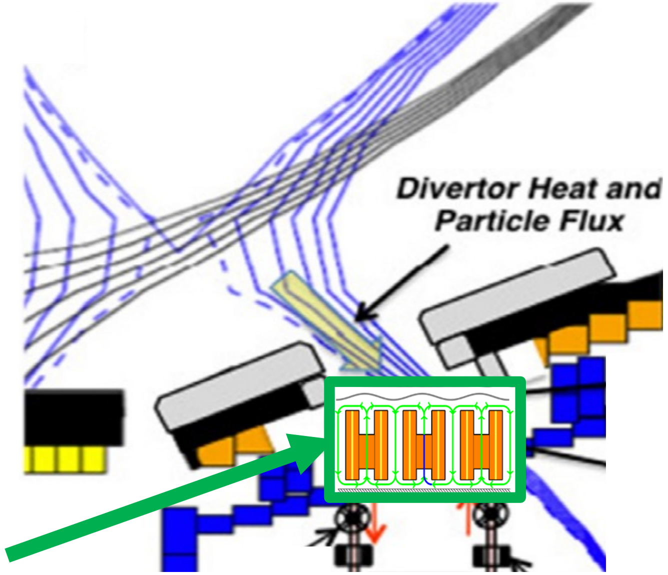
Experimental, analytical, and numerical validation

Divertorlets

- MHD Drag scales with U and B^2
  - Very difficult at reactor conditions
  - Need high speeds over a small area
- Slow flow only possible with small L
- Divertorlets with a radiator like flow allows slow flow to take large q!
  - *No moving parts*

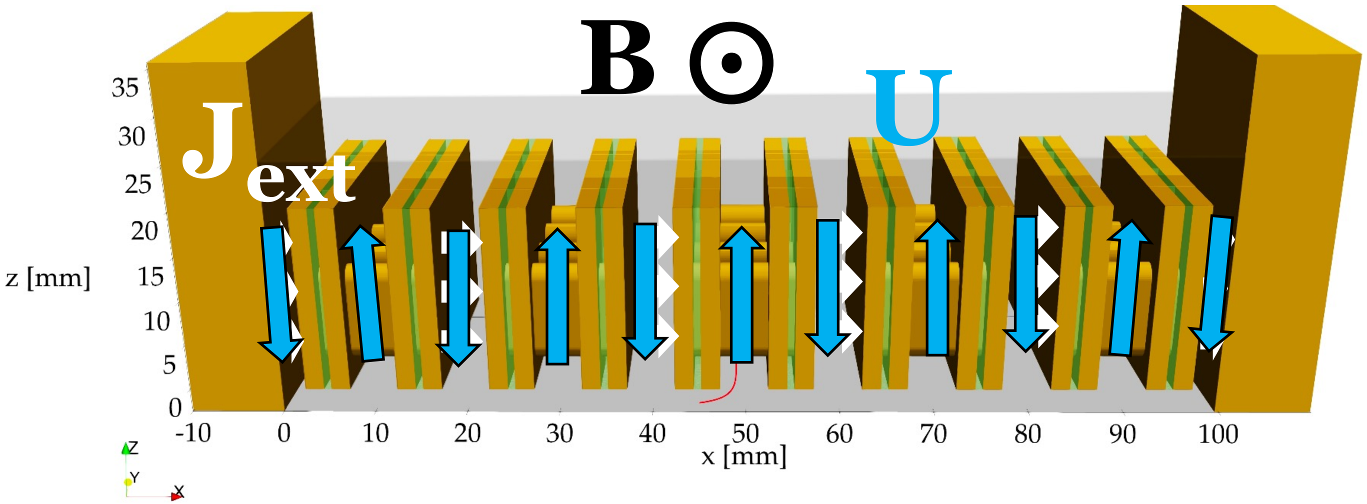


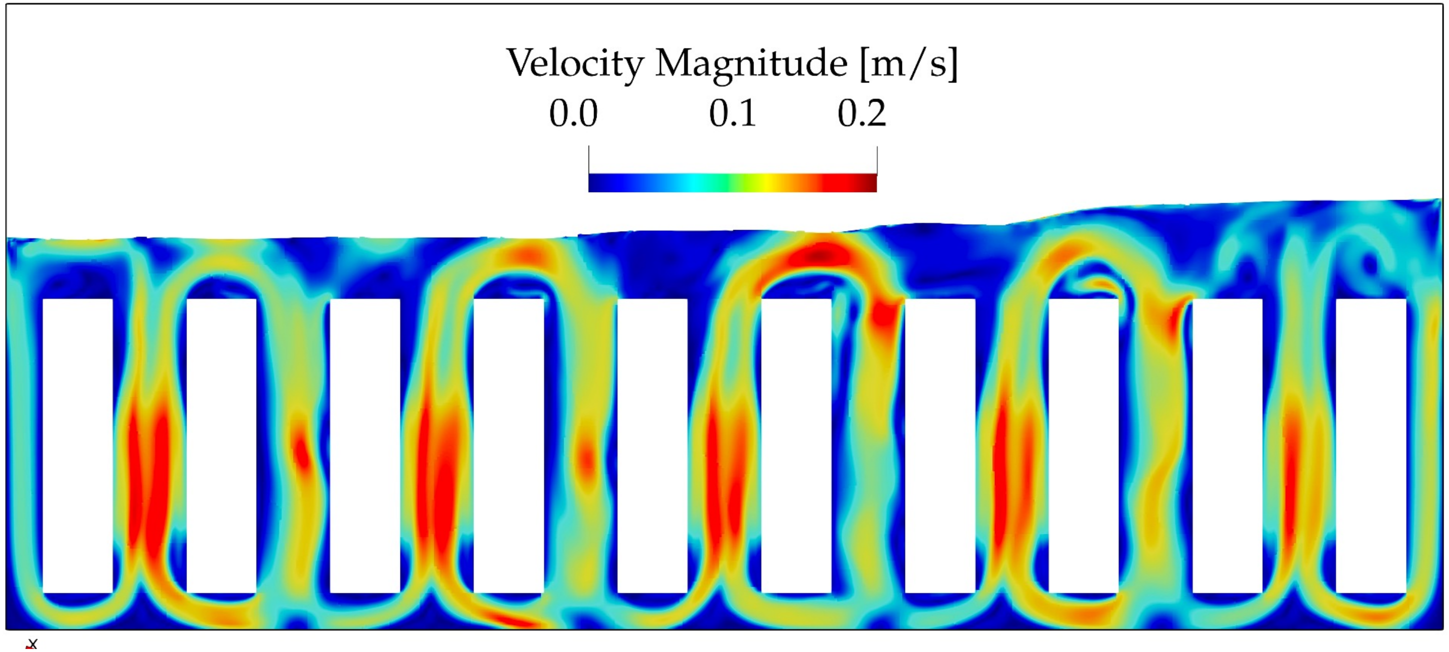




Plasma Control

Explanation of how it works

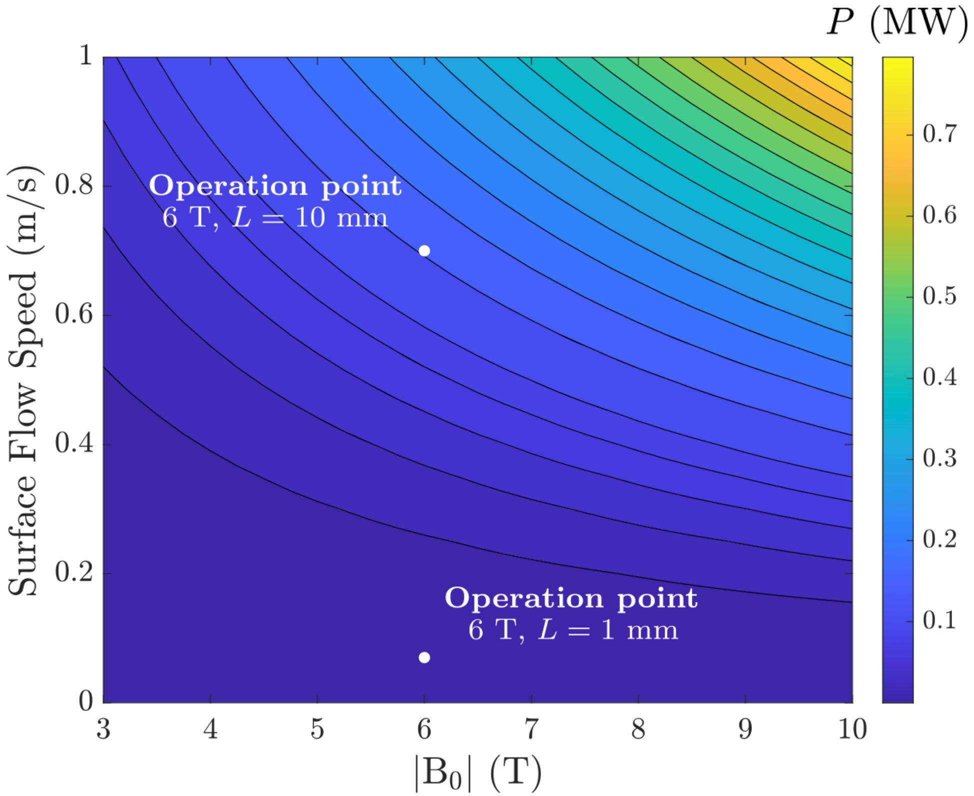




Plasma Control

Divertorlets Power Requirements

- <1% of 500 MW DEMO
- Large improvement compared to JxB thrust



Reactor scale projections for 10 MW/m²

LEAP (Lithium Experiment Application Platform)

In development at PPPL

Lithium Experimental Application Platform (LEAP)

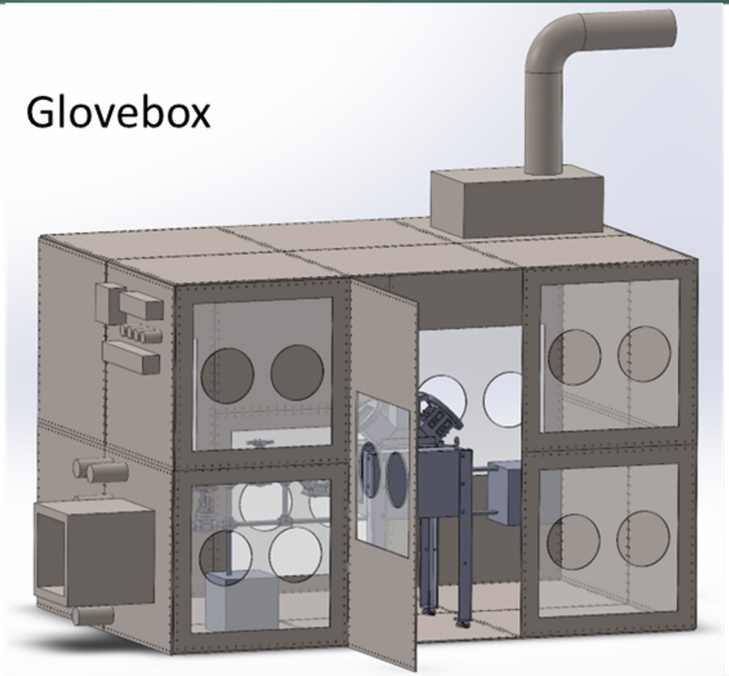
For testing full sectors of fast-flowing lithium systems and LMPFCs with heat sources and magnetic fields

Designed to handle 50lb of liquid Li.
Planned to be largest working liquid Li experiment in the US.

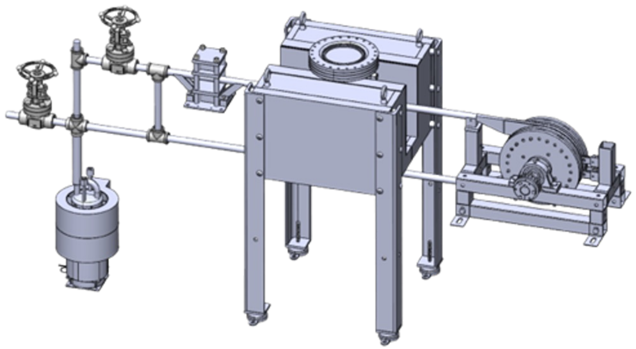
Central component is a large (2m x 3m x 2m) prefabricated modularized glove box.

Argon purging during operation (H_2O/O_2 level <1000 ppm) to ensure safety and inert environment.

Equipped with inflatable gaskets and quick-open door for easy access and maintenance between operations.

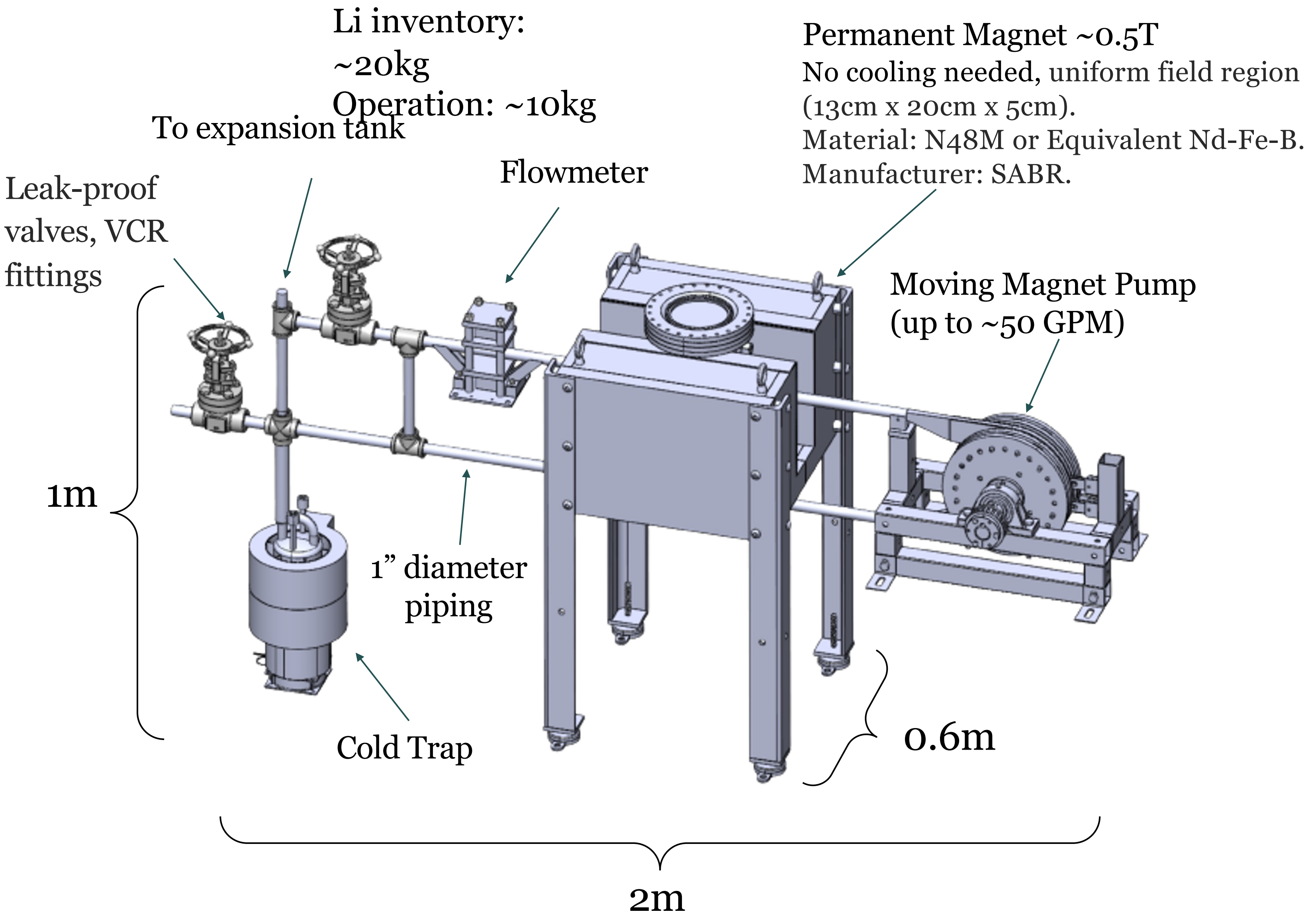


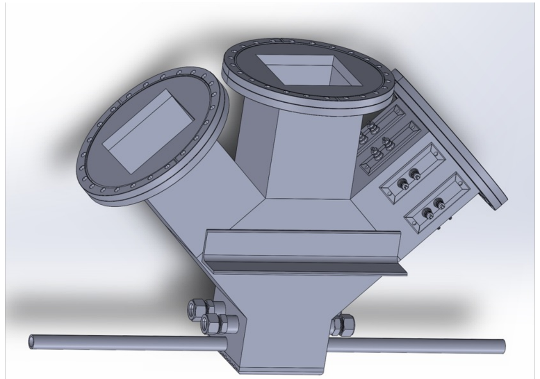
Li Loop Apparatus



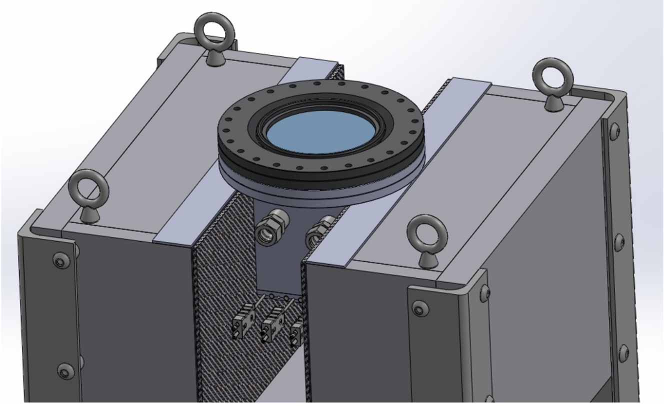
Manufacturer samples

Li Loop Apparatus in LEAP





Alternative angled diagnostic flanges



Cartridge heaters &
adjustable mounting provisions

Conclusion

- **Future work**
  - Magnetic centrifuge for LiH extraction
  - FreeMHD
    - Heat transfer/Seebeck coefficient, Addition of thermal gradients
    - Surface tension modeling
    - B Induction (extreme conditions where induced B is required, e.g. General Fusion)
  - Possible project on LTX droplet movement

Plasma Control
control.princeton.edu

Funding Acknowledgments

- US DOE Field Work Proposal No. 1019 (Domestic Liquid Metal Plasma Facing Component Development)
- LDRD (Laboratory Directed Research Development) Project No. PPPL-128 (Divertor Design for Low-Recycling Regime Tokamak)

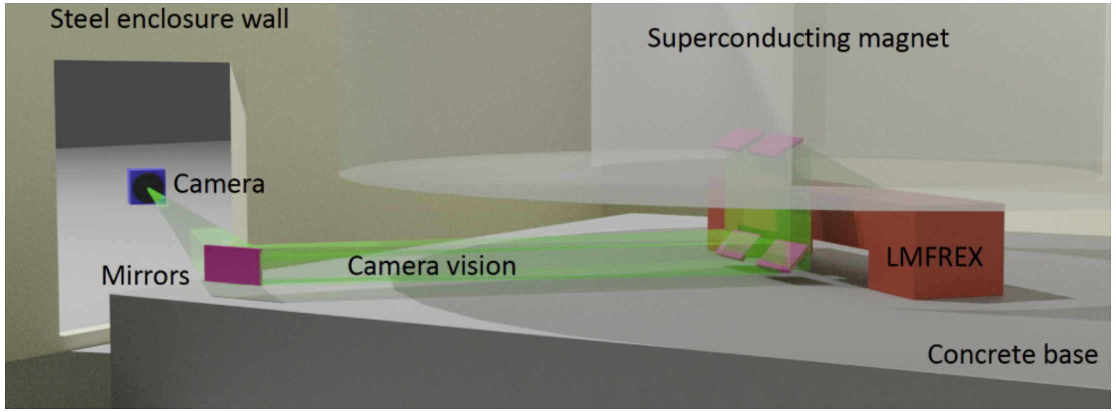
Additional Slides

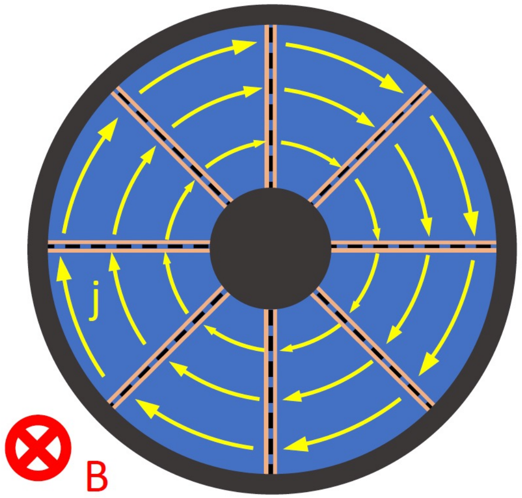
JxB Thrust

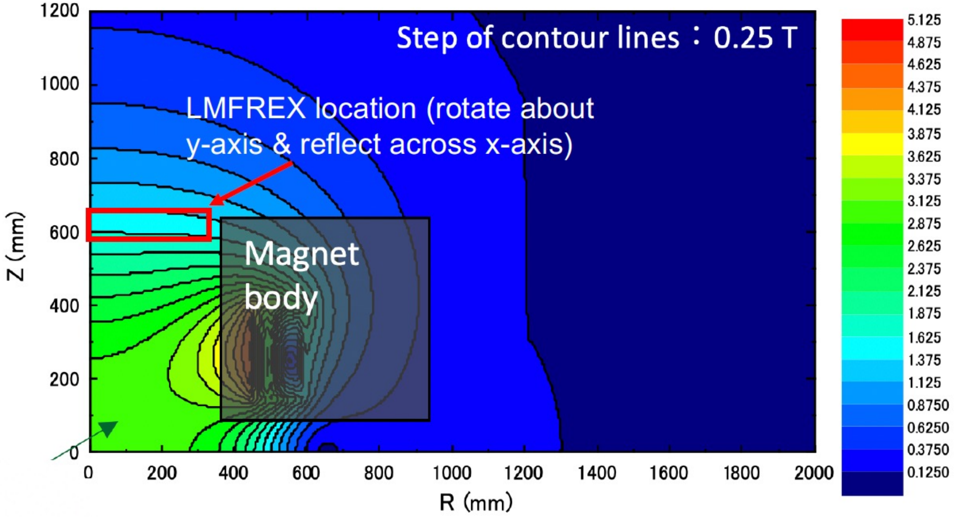
brianwynne@princeton.edu

Oroshhi-2/NIFS/Kyoto University Collaboration

Additional Details







Plasma Control

Critical flow speeds

- Critical flow speed, 1-20 m/s

$$v_{\mathrm{cr}} = \frac{4\alpha L}{\pi}\left(\frac{q_{\perp}}{k\Delta T_{\mathrm{cr}}}\right)^2$$

Additional Slides

LEAP

brianwynne@princeton.edu

Lithium Experimental Application Platform (LEAP)

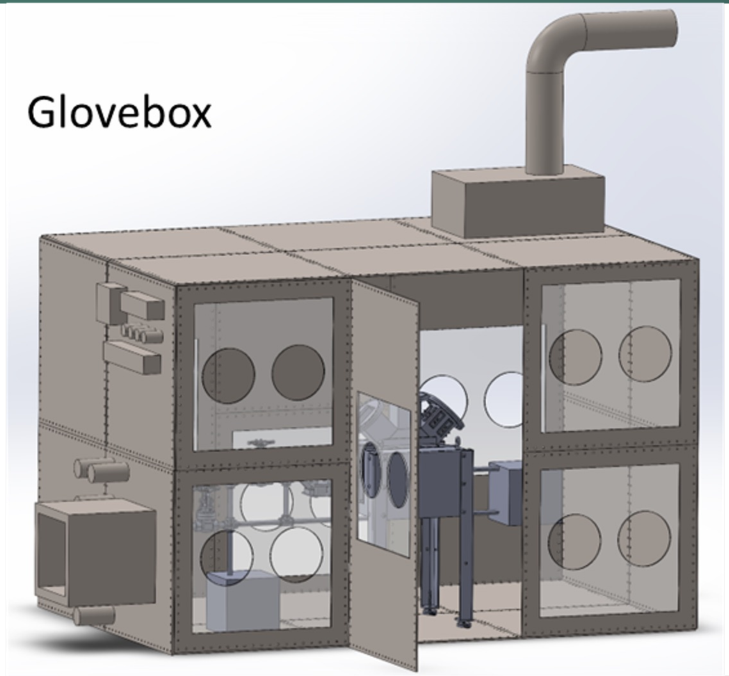
For testing full sectors of fast-flowing lithium systems and LMPFCs with heat sources and magnetic fields. Planning phase.

Designed to handles 50lb of liquid Li. Largest working liquid Li experiment in the US.

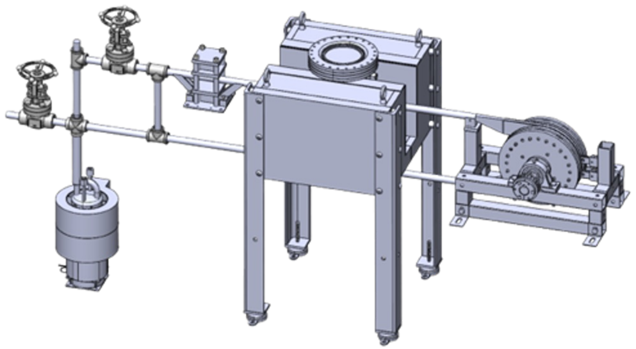
Central component is a large (2m x 3m x 2m) prefabricated modularized glove box.

Argon purging during operation (H_2O/O_2 level <1000 ppm) to ensure safety and inert environment.

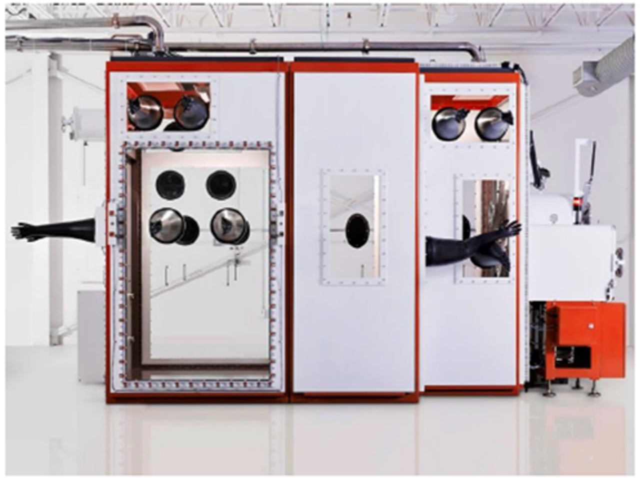
Equipped with inflatable gaskets and quick-open door for easy access and maintenance between operations.



Li Loop Apparatus

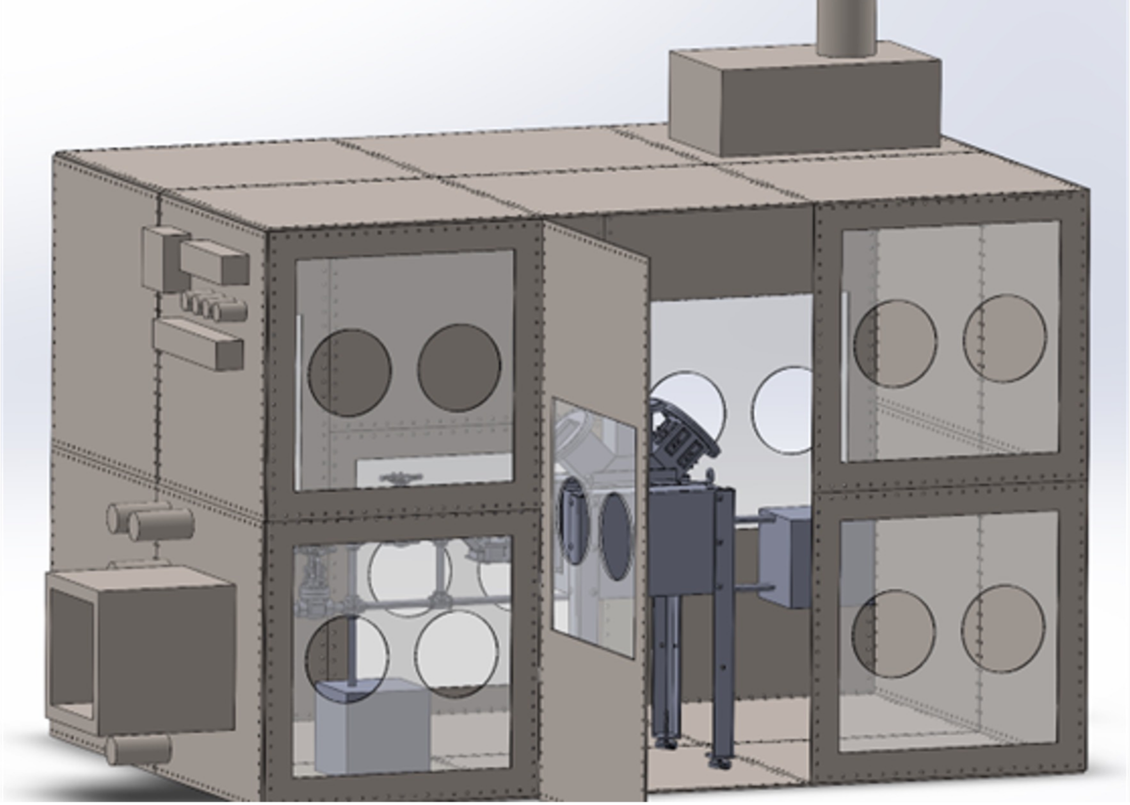


Manufacturer samples

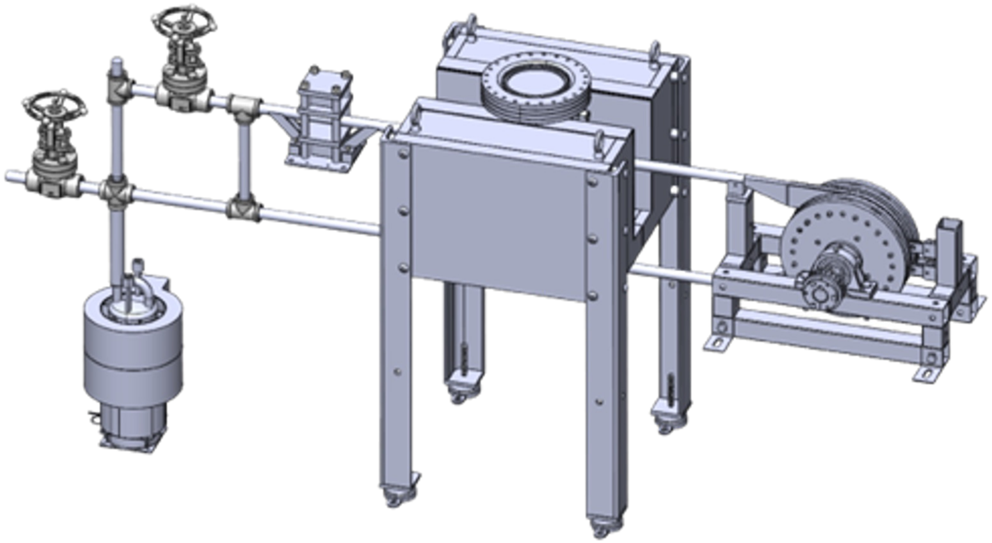


LEAP Plans

Glovebox



Li Loop Apparatus



Plasma Control

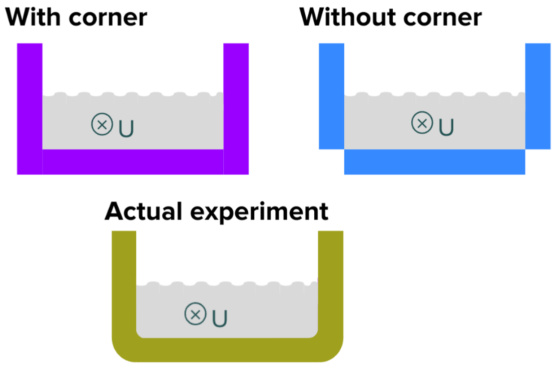
Additional Slides

LMX-U

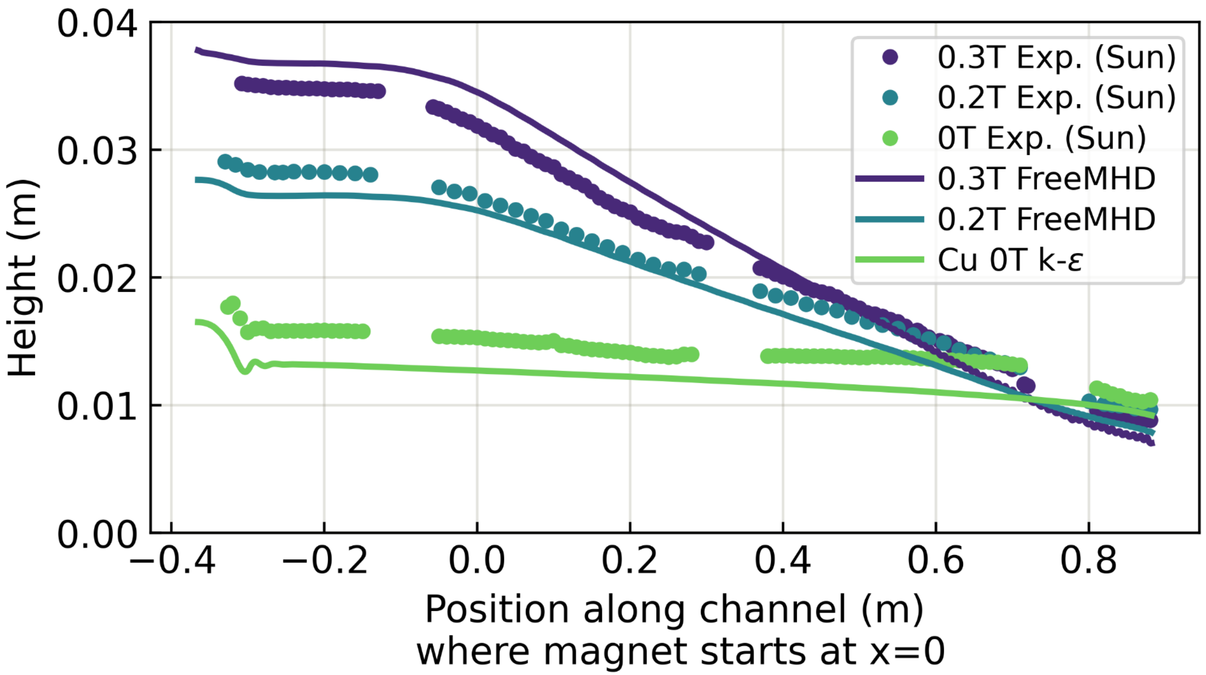
brianwynne@princeton.edu

Plasma Control

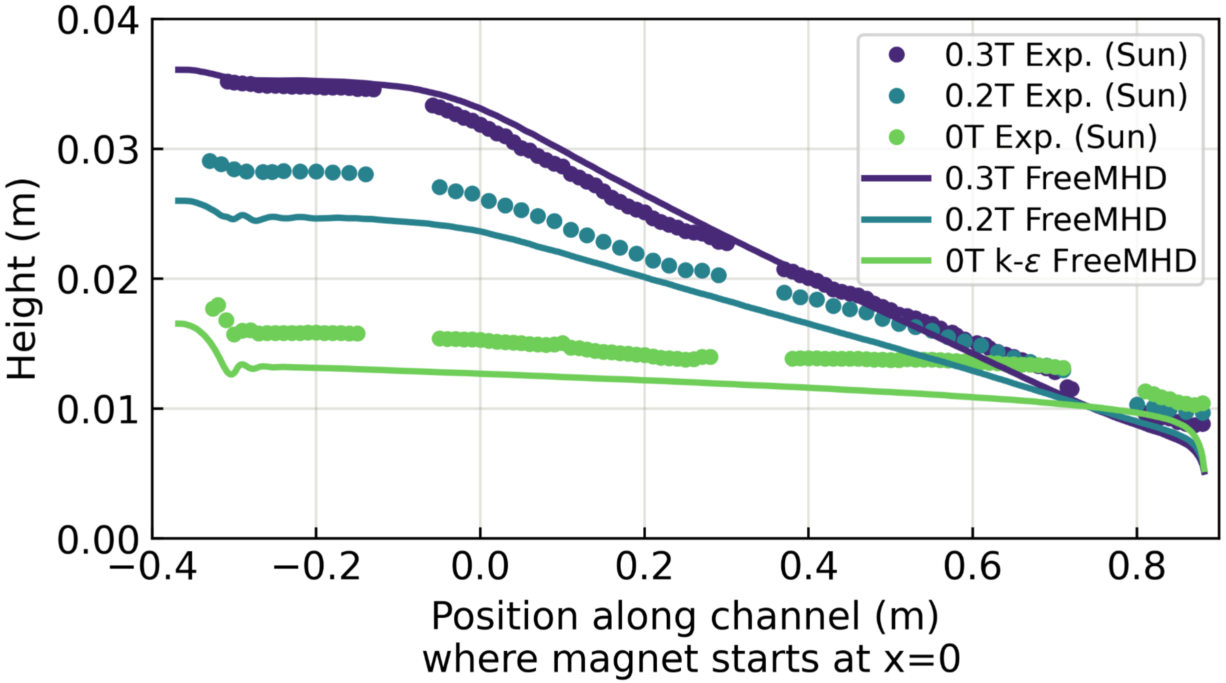
Liner: Experimental Differences



With Corner

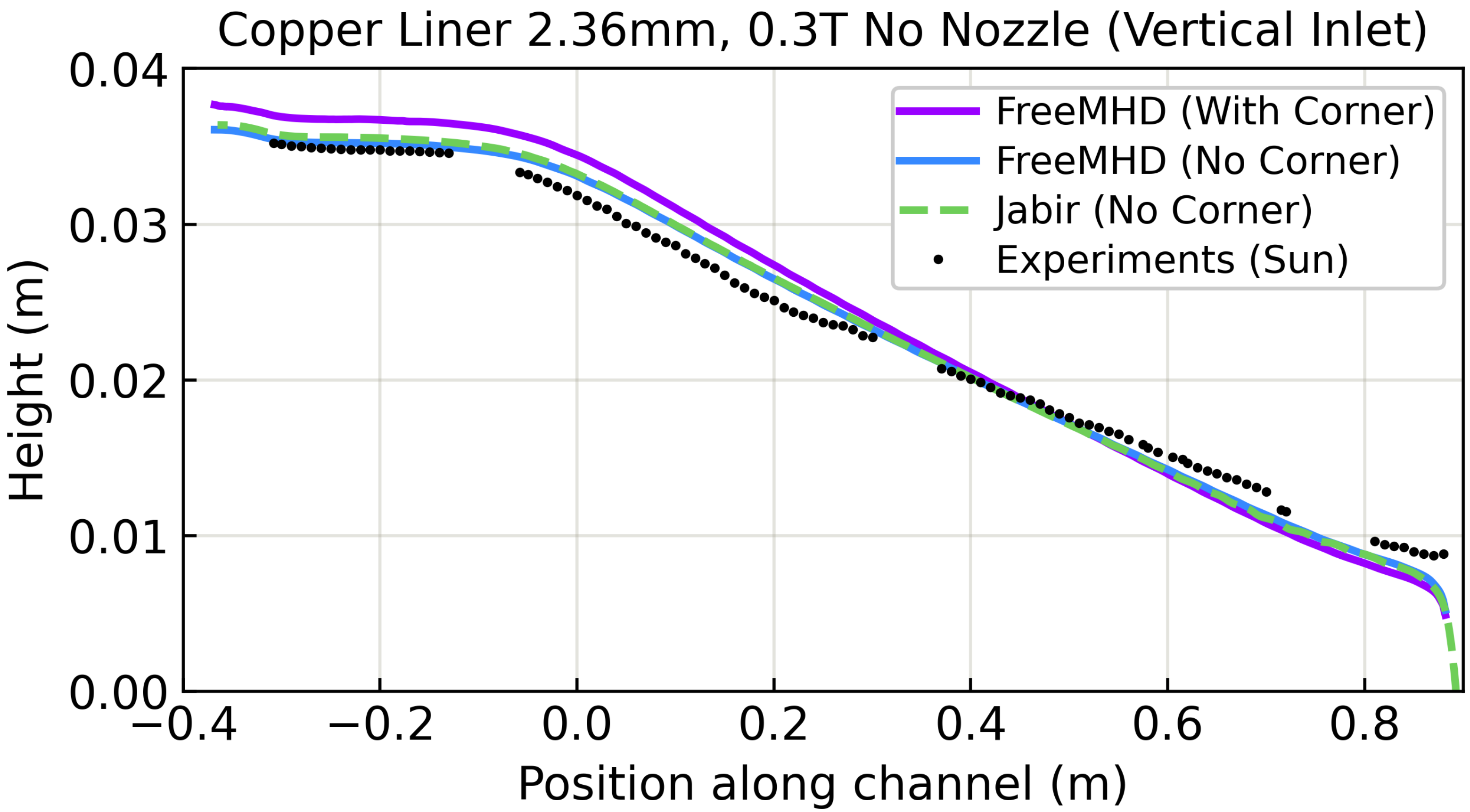


No Corner



Plasma Control

Liner: Corner vs No Corner



- Copper 2.36mm Liner
- 0-0.3T
- Vertical inlet (No nozzle)

Plasma Control

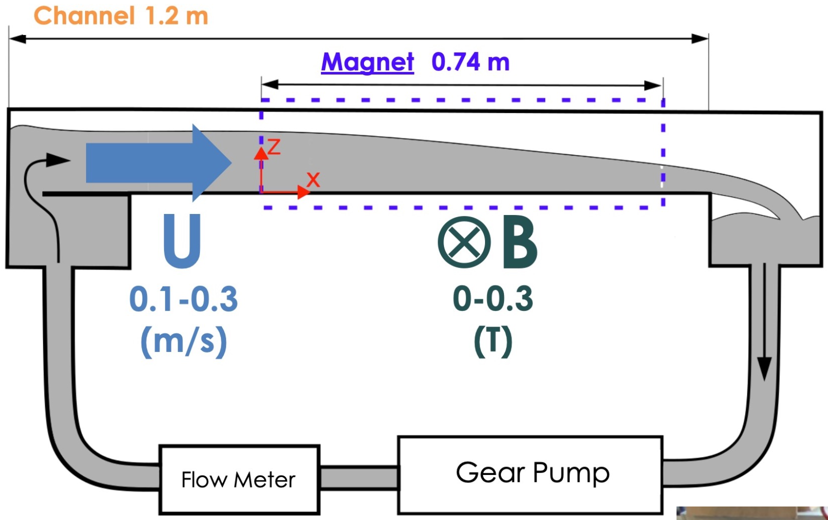
LMX - Methods

LMX-U

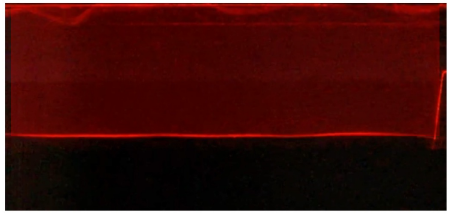
- Flow loop, free surface channel
- Test bed for liquid metal experiments
- Galinstan (gallium, indium, and tin)

Measurement Methods

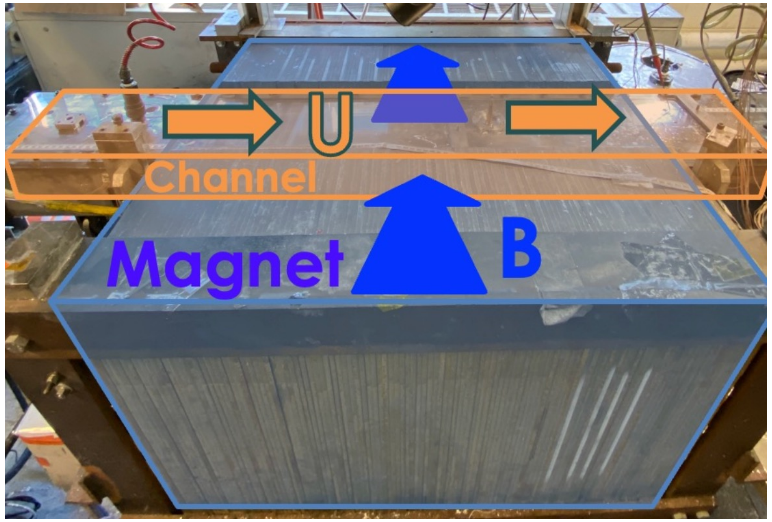
- Heights of the liquid metal
- Particle tracking of the surface
- Electric potential readings



Top View of Channel



Laser line on surface



Plasma Control

Free Surface Velocity

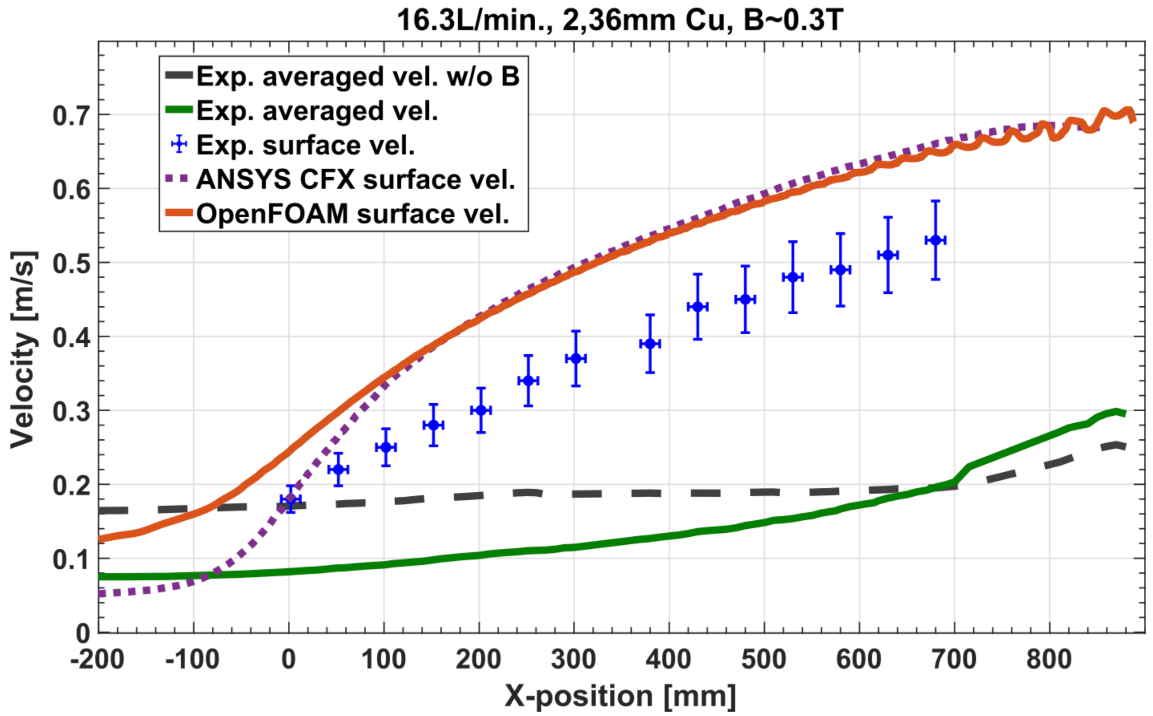
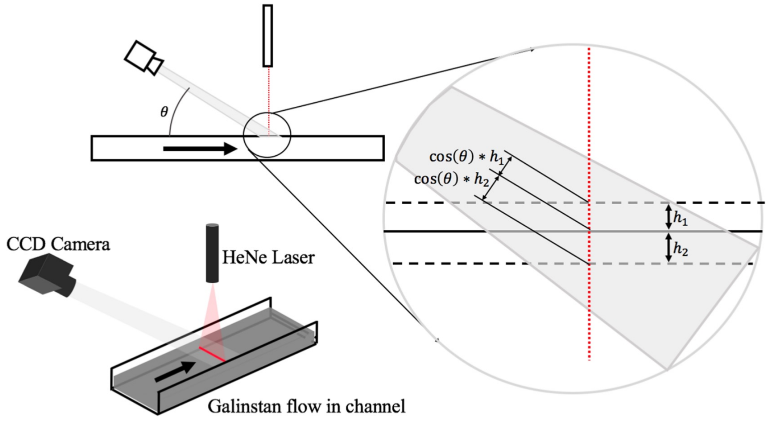


Figure 9. Surface and average velocity evolutions obtained from LMX-U experiments and simulations. The experiment setup is the same as in figure 5 with 0.3 T.

Plasma Control

Liquid Metal R&D without plasmas

- **Liquid Metal eXperiment Upgrade** (LMX-U)
  - Height-adjustable nozzle at inlet allows inlet depth to be changed:
    - Max. flow speed: 2 m/s
    - Removable nozzle
  - Channel liner: acrylic base. Width: 109 mm
  - Inclination angle range: 0˚ - 7˚.
  - Movable channel.
- **Diagnostics**
  - Laser sheet for depth measurements.



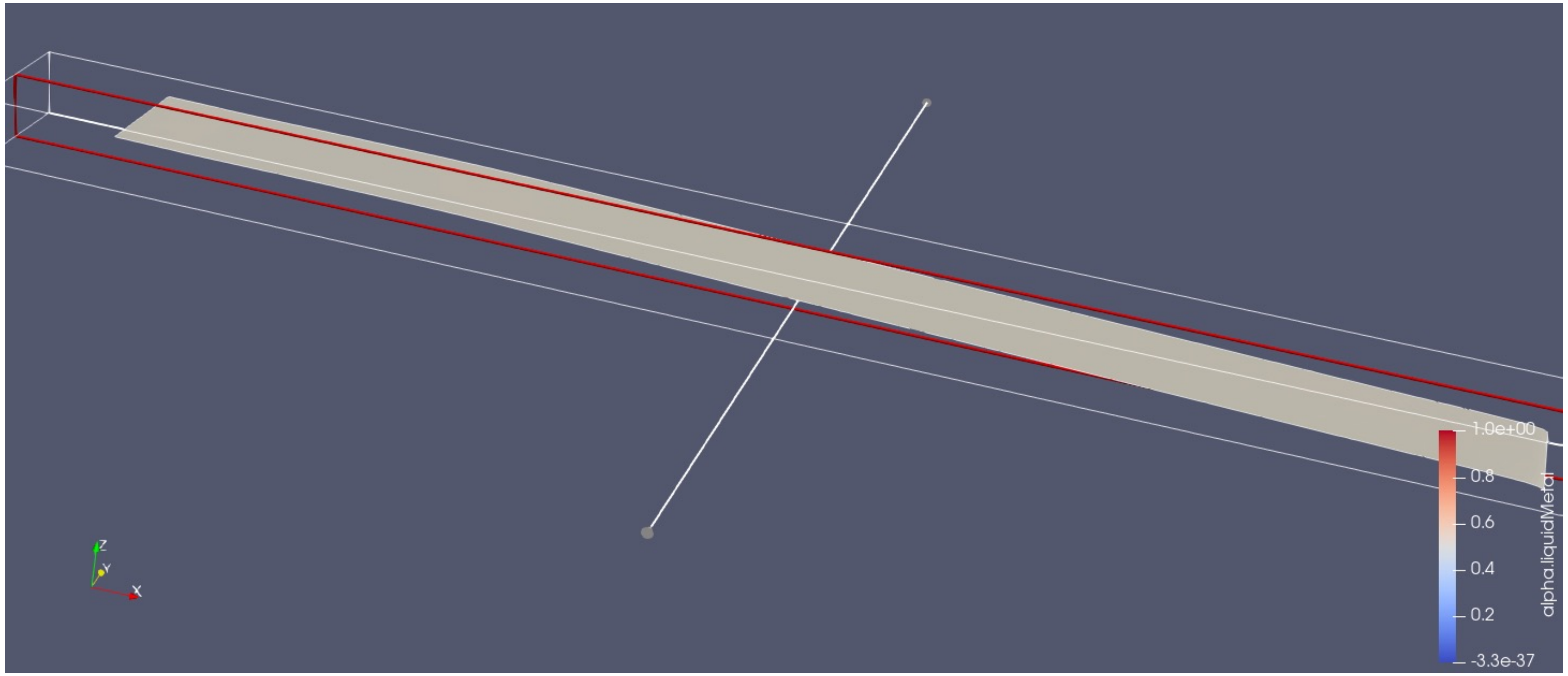
Hvasta [NF, 2018]

Plasma Control

Sim vs LMX (Additional)

Source of data from sims, plotting:

- Contour (alpha=0.5), Slice Along Flow Direction (Y Normal)



Plasma Control

Hydrualic Jump

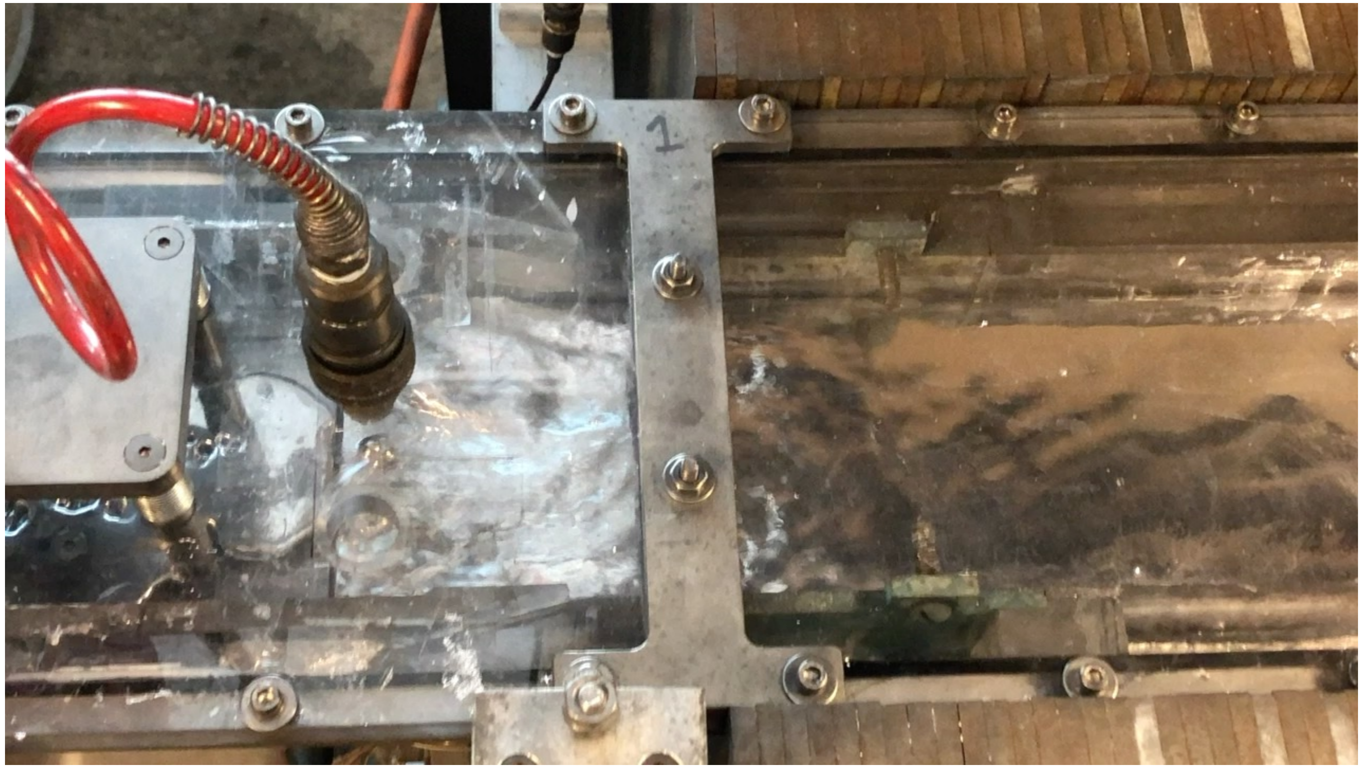
Froude number

When:

Fr = 1, critical flow,

Fr > 1, supercritical flow (fast rapid flow),

Fr < 1, subcritical flow (slow / tranquil flow)



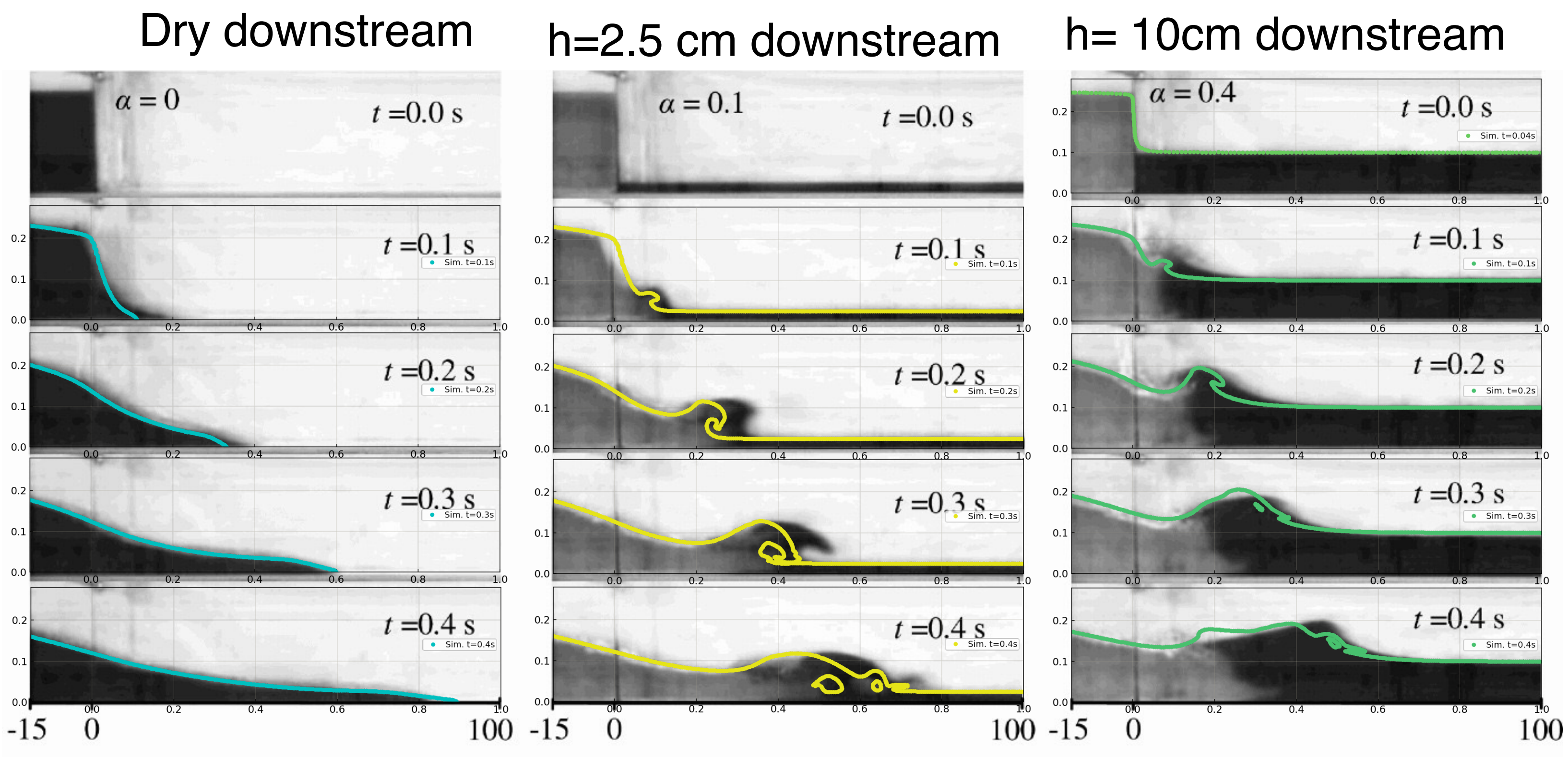
Plasma Control

Additional Slides

Dam Breaking

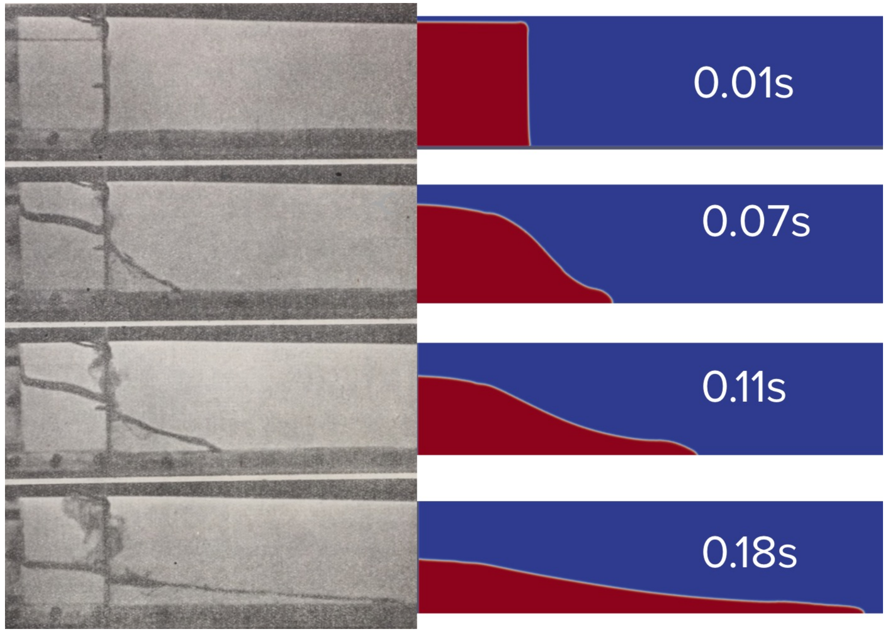
brianwynne@princeton.edu

Dam Breaking (Move to Backup)

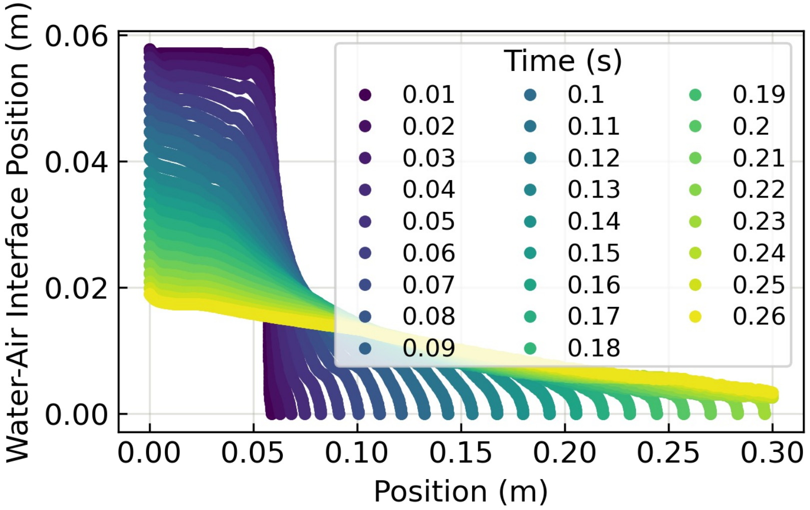


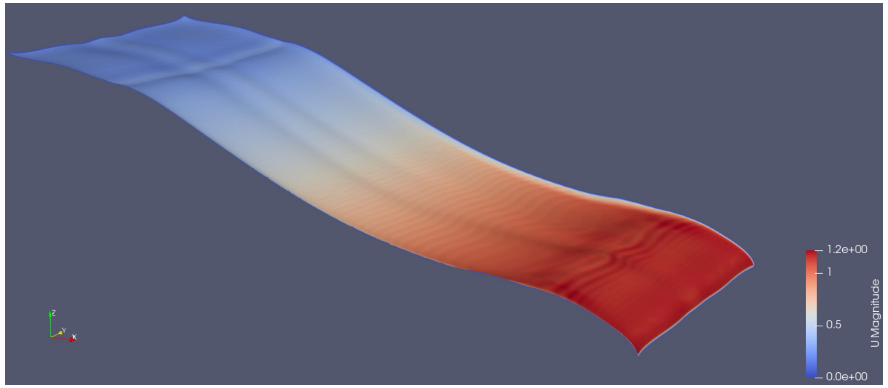
[1]

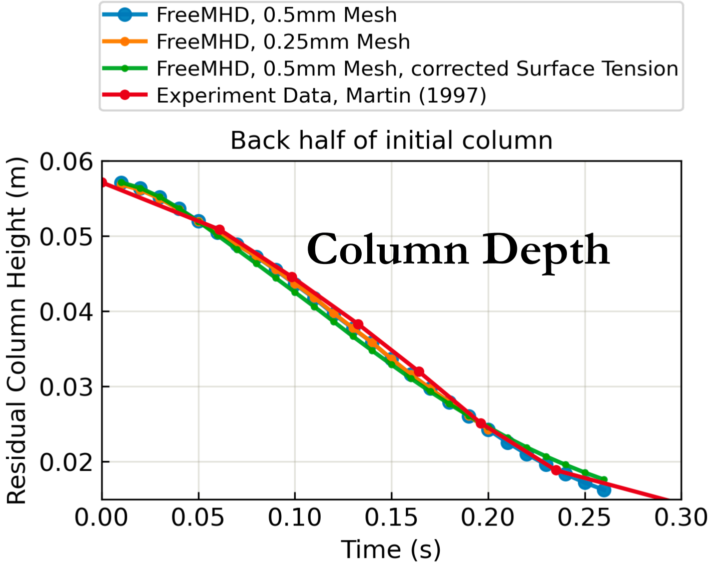
Dam Breaking (Martin 1952) Backup

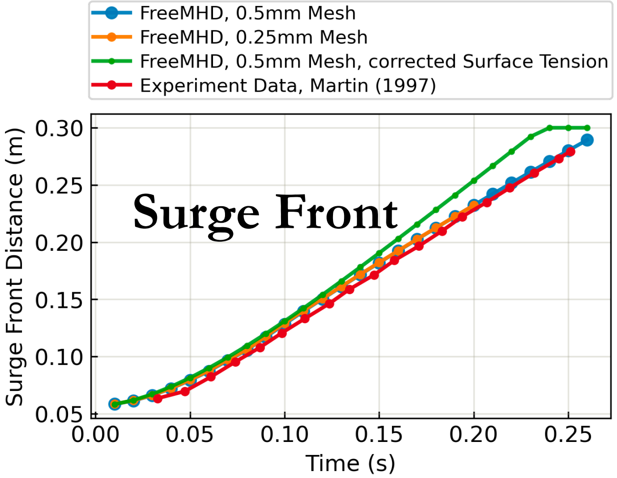


- Experiments using water [1]
- Tracking column depth and surge front





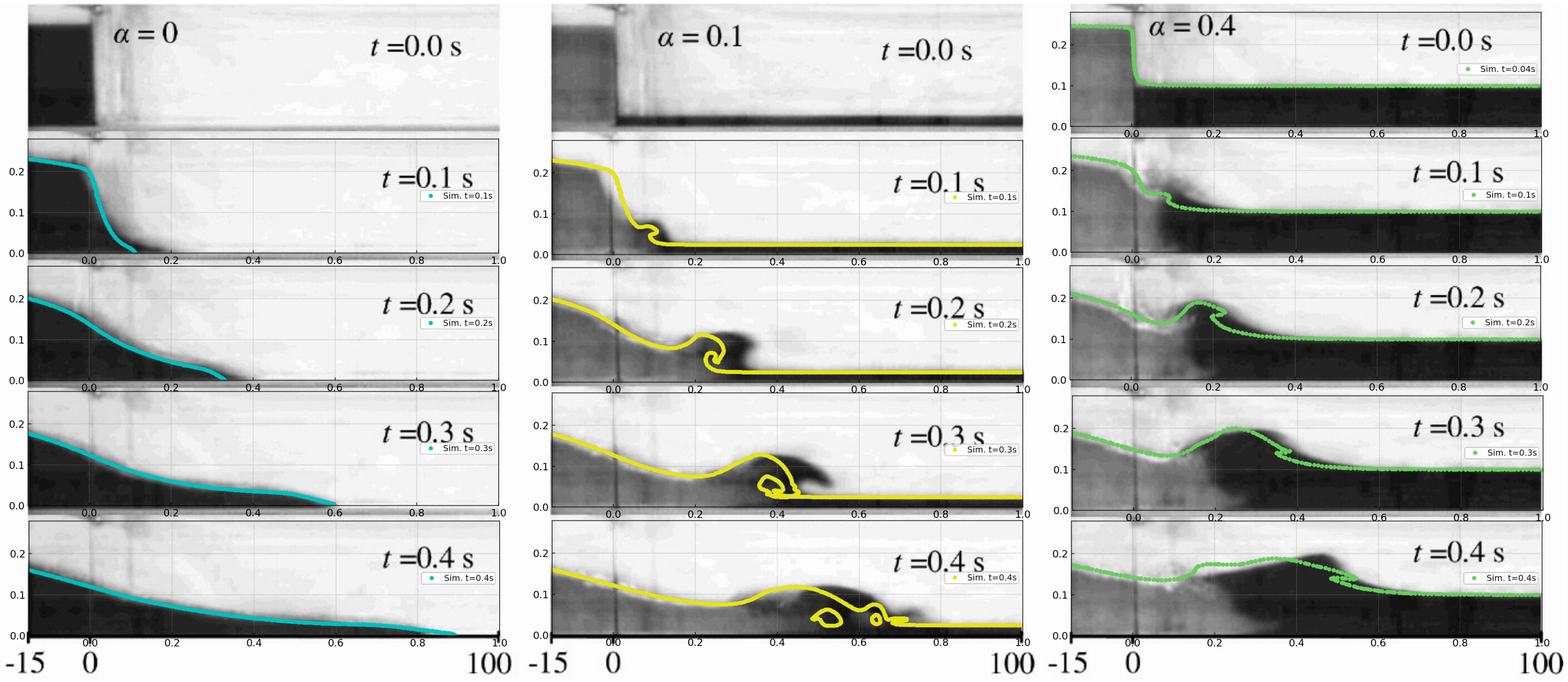


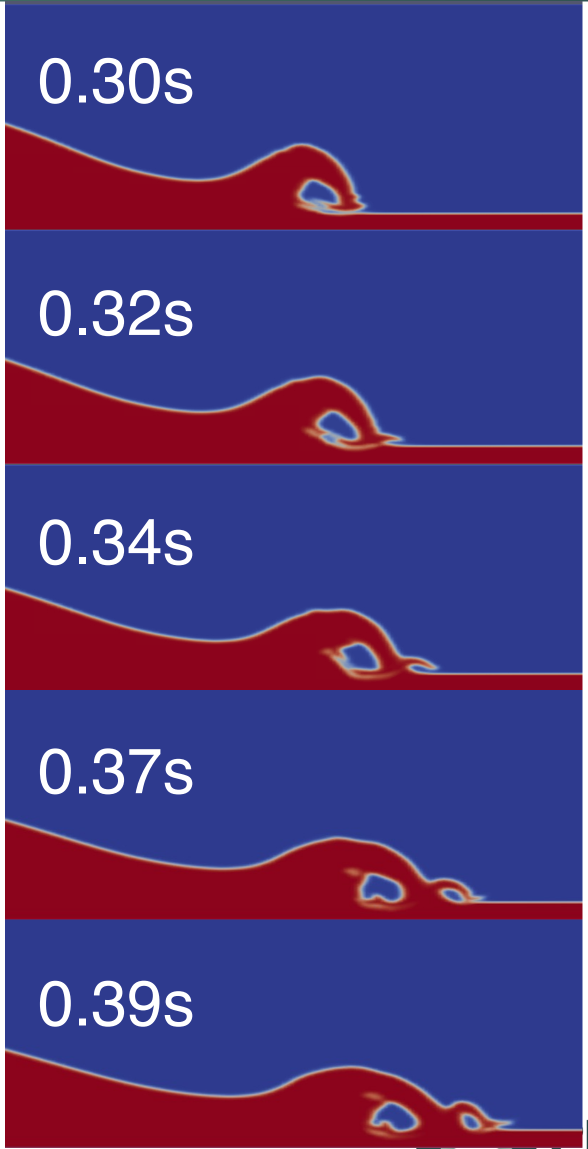


Plasma Control

[1] Martin, J.C. (1952). Part IV. An experimental study of the collapse of liquid columns on a rigid horizontal plane.

Dam Breaking (Additional)





"While the MULES scheme performs well when the interface deformation is moderate, it struggles to maintain a sharp interface with controllable thickness in cases with complicated free surface shapes.
Furthermore, blobs of smeared interfaces with $0 < \alpha < 0.5$ can be carried away from the interface, into the heavier fluid, especially when there is a high velocity component normal to the interface. These blobs accumulate, leading to the formation of nonphysical bubbles that may disturb the flow."

Plasma Control

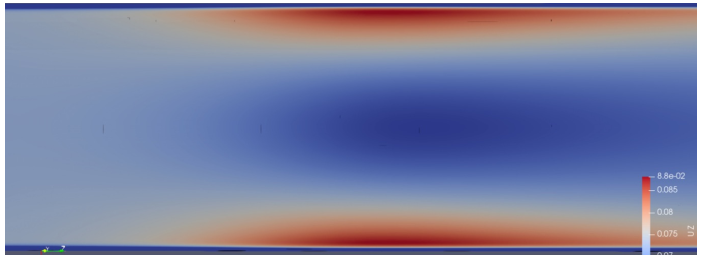
Additional Slides

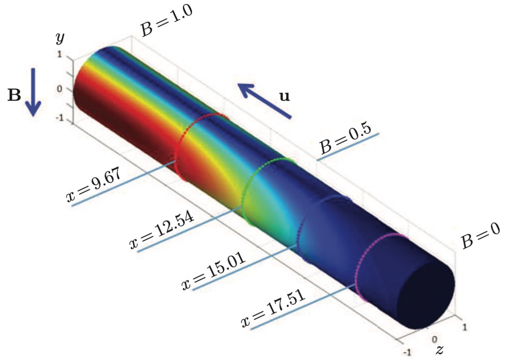
Fringing B

brianwynne@princeton.edu

Fringing B Field

- Phi_max **6.35463e-4**?
- U0=0.069, B0=0.21, L=48.59e-3
- phi/(U0 L B0)=6.35463e-4/(0.069*48.59e-3*0.21)=0.9025 (thec
  match with theory would be **6.548e-4**
- Pressure normalized by σULB² = 2.878e6*0.069*48.59e-3*0.21^
- dP_nondimen = σUB² = 2.878e6*0.069*0.21^2=8,757.4662
- dP_fd = 0.0678?
- (759/8,757.466)/.0678=1.2783024525





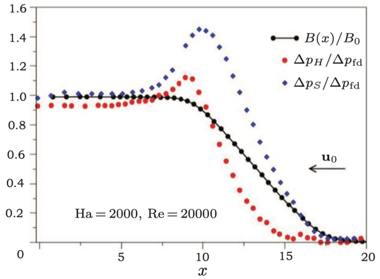


Fig. 5. Pressure gradients normalized by the asymptotic value for fully developed flow $dp_{fd}/dx = 0.0678$, as a function of the axial x-coordinate for Ha = 2000, Re = 20000.

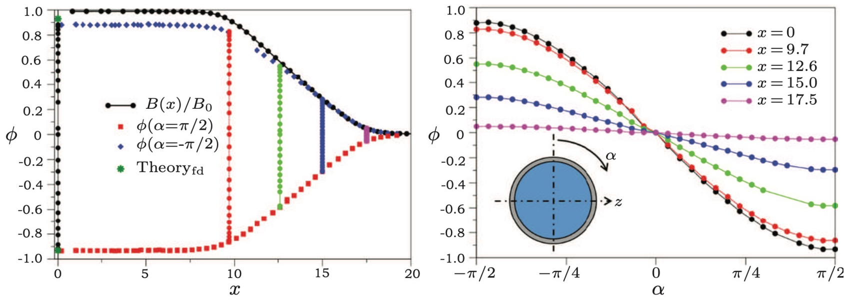
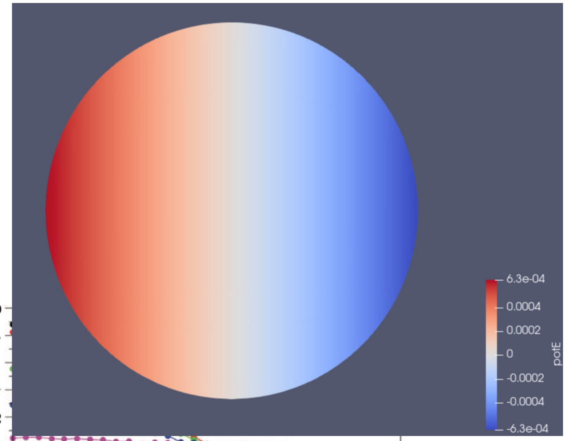


Fig. 7. Non-dimensional electric wall potential at Ha = 2000, Re = 20000. Left: potential $\phi(x, \alpha = \pm\pi/2)$ at both sides along the pipe; right: wall potential as a function of the circumferential angle α around the pipe at different axial positions, as indicated in the left graph.



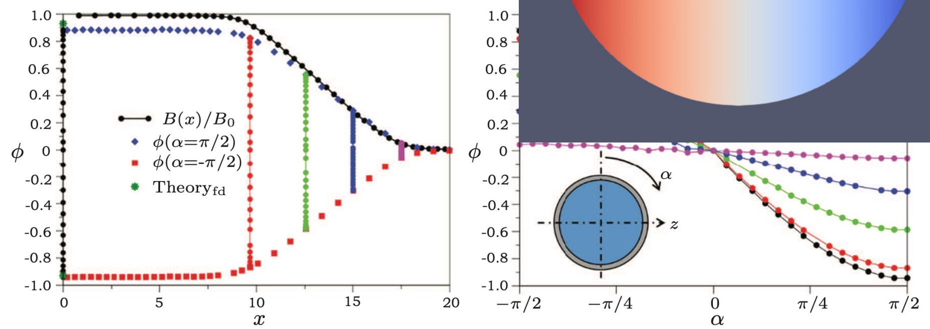
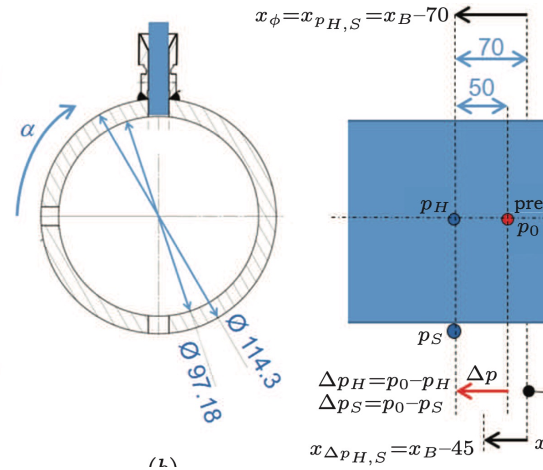


Fig. 8. Non-dimensional electric wall potential at Ha = 5500, Re = 10000. Left: potential $\phi(x, \alpha = \pm\pi/2)$ at both sides along the pipe; right: wall potential as a function of the circumferential angle α around the pipe at different axial positions, as indicated in the left graph.



Plasma Control

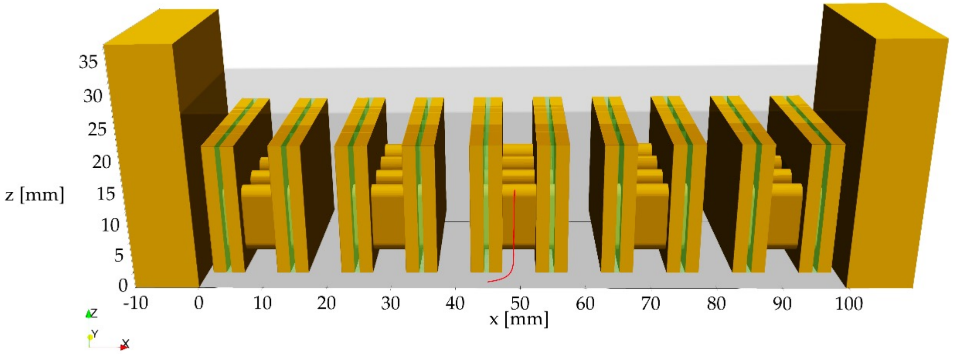
Additional Slides

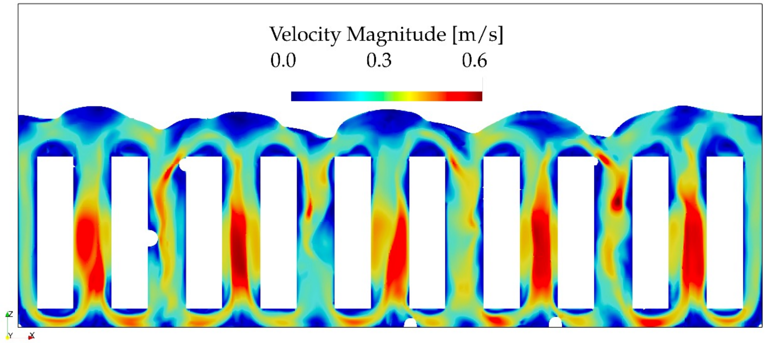
Divertorlets

brianwynne@princeton.edu

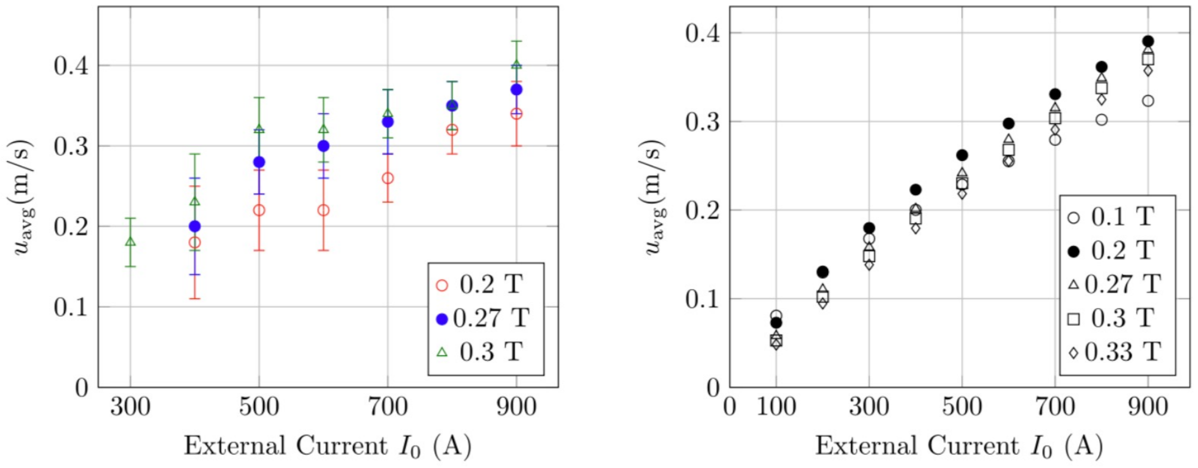
Issues and solutions: Divertorlets

- F_MHD scales as U and B^2
- Very difficult at reactor conditions
- Need high speeds over a small area… Divertorlets



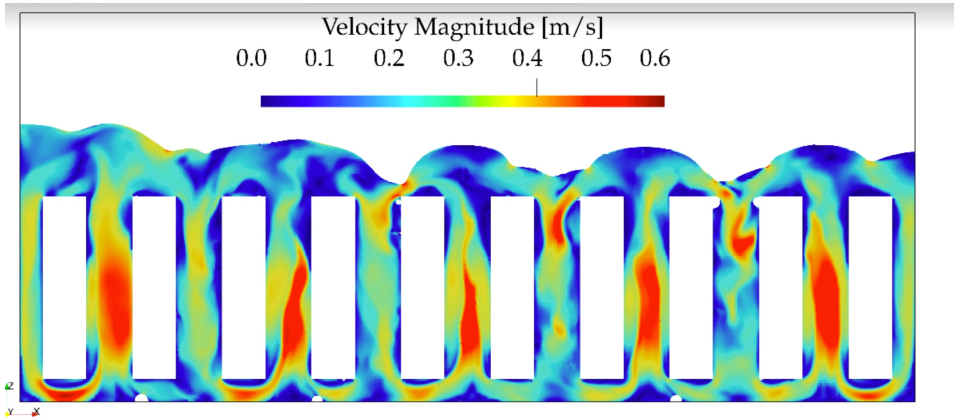


Divertorlets



(a) Experimental results (b) Simulation results

Figure 9. Results from experiments and simulations for a Cu–GaInSn divertorlets prototype.

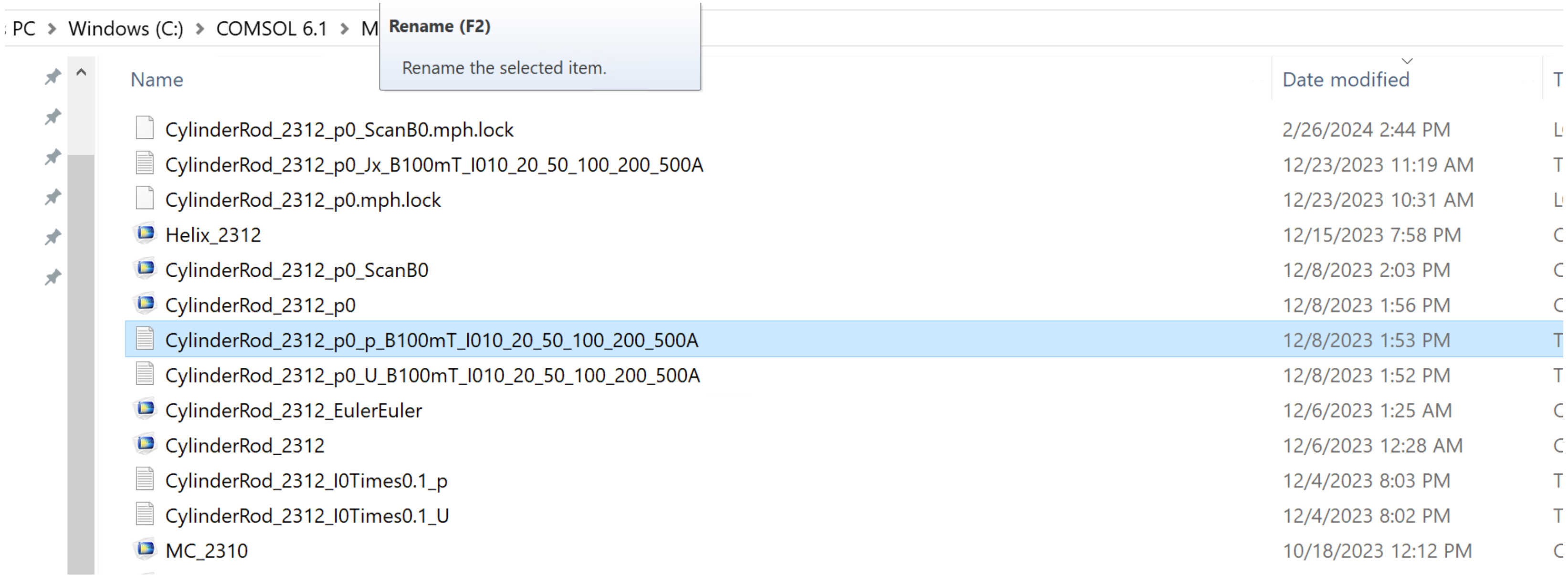


igure 5.40: Instantaneous velocity and free surface shape for the case of $I_0 = 800\,A, B = 0.2T$ using an initial filling level of $\frac{1}{2}w_{ch}$ (top) and w_{ch}(bottom).

Plasma Control

Centrifuge

- “recent” comsol sims?



Plasma Control

FreeMHD Background

- Previous attempts at modeling
  - Codes to simulate the behavior of **free-surface** liquid metal (LM) under fusion-relevant conditions are not readily available
  - Mainly steady-state, 2D, or simplified models for internal flows [1,2]
- FreeMHD
  - Developed by Jabir Al-Salami
  - Free surface liquid metal magnetohydrodynamics (MHD) solver

[1] N. B. Morley, S. Smolentsev, R. Munipalli, M.-J. Ni, D. Gao, and M. Abdou. "Progress on the modeling of liquid metal, free surface, MHD flows for fusion liquid walls." In: 72 (2004), pp. 3–34. doi: 10.1016/j.fusengdes.2004.07.013

[2] S. Smolentsev and M. Abdou. "Open-surface MHD flow over a curved wall in the 3-D thin-shear-layer approximation." In: Applied Mathematical Modelling 29.3 (Mar. 2005), pp. 215–234. doi: 10.1016/j. apm.2004.07.002

Plasma Control

Divertorlets next steps

- Thin slats
- Round corners of slats
- Increase depth?
- Want to reduce amplitude of waves
- AC instead of DC?

Plasma Control

Additional Slides

FreeMHD

brianwynne@princeton.edu

Surface Tension

- surface tension force acting on the interface between the two phases
  - modeled using the continuum surface force (CSF) method
  - the surface tension force is given by $$\boldsymbol{F}_{st} = \sigma_{st}\kappa\nabla\alpha$$
  - curvature on the interface between the two fluids $$\kappa = \nabla \cdot \left[\frac{\nabla\alpha}{|\nabla\alpha|}\right]$$

Plasma Control

Backup slide: computational

Case	CPU Hours	Time to run	Cell size	Max Time Step
Closed Channel	~12	1 hr	1-1000μm	0.01-1ms
Free Surface, Dam Breaking	47.3	1.3 hr	500μm	1 ms
Free Surface, LMX	1865	12 hr	500μm	0.2ms

Plasma Control

Parameters

Case	Re	Ha	N
Closed Channel			
Free Surface, Dam Breaking			
Fringing B			
Free Surface, LMX			

Plasma Control

FVM and VoF (additional information)

- Finite-volume (FVM) OpenFOAM framework

- Volume of fluid (VoF)
  - MUlti dimensional Limiter for Explicit Solution (MULES) method
    - modifies the advection of the volume fraction by adding an interface compression velocity term

Plasma Control

MULES (additional information 1)

- MUlti dimensional Limiter for Explicit Solution (MULES)
  - Modifies the advection of the volume fraction by adding an interface compression velocity term
  - Controls thickness and reduces smearing of interface

1. Integrate change of the volume fraction
2. Discretize with implicit Euler time-stepping*

    - Despite its first order accuracy, this scheme's stability that allows taking large time-steps is conducive for carrying out efficient simulations of transient phenomena
    - λ_m is 1 in the vicinity of the interface, and 0 elsewhere

$$\int_V \frac{\partial \alpha}{\partial t} dV + \int_V \nabla \cdot (\alpha \boldsymbol{U}) dV = 0$$

$$\frac{(\alpha)_c^{(t+1)} - (\alpha)_c^{(t)}}{\Delta t} = -\frac{1}{V_c} \sum_{f=1}^{N_f} (F_u + \lambda_m F_c)^{(t)}$$

Plasma Control

MULES (additional information 2)

- Van Leer scheme used for the advection of α
  - second order accuracy, stability and low numerical diffusion
- limiting function $\boldsymbol{\Psi}(r_f)$
  - where r_f is defined as

$$r_f = \frac{2(\nabla \boldsymbol{u} \cdot \boldsymbol{d}_{CN}) \cdot (\boldsymbol{u}_N - \boldsymbol{u}_C)}{|\boldsymbol{u}_N - \boldsymbol{u}_C|^2}$$

$$\Psi(r_f)_{\text{Van-Leer}} \quad \frac{r_f + |r_f|}{1 + r_f}$$

[1] M. S. Darwish and F. Moukalled. “TVD schemes for unstructured grids.” In: 46 (2003), pp. 599–611. doi: 10.1016/s0017-9310(02) 00330-7

Plasma Control

Additional Slides

Tungsten Mesh

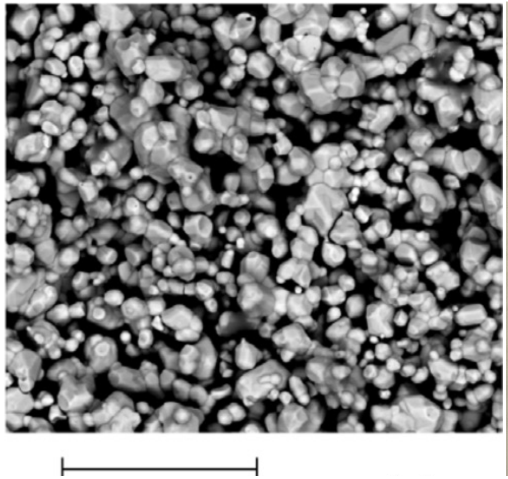
brianwynne@princeton.edu

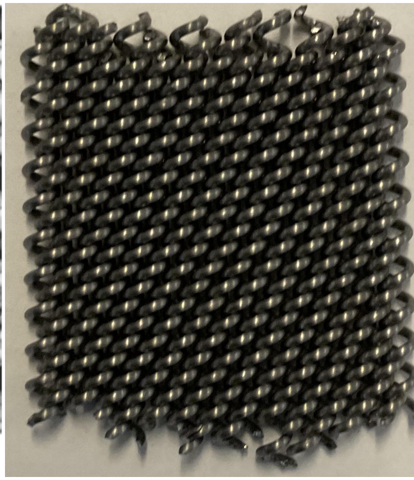
Plasma Control

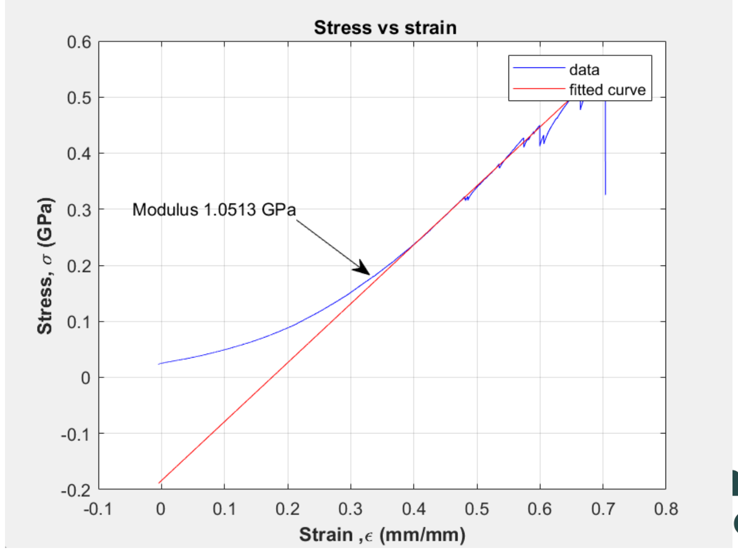
Tungsten Mesh for Splash Free Liquid Lithium

- Tungsten mesh to prevent droplet ejection
- Studying different pore size for the mesh
- **New flexible sawn mesh (like Mithril)** may avoid breakage! Studying:
  - Material properties (elasticity, stress strain, …)
  - Different mesh structures
- Studying different mesh options for
  - Evaporative LM system
  - Injection of currents in LM flows
  - …etic fields

← Mithril (Lord of the Rings)







Additional Slides

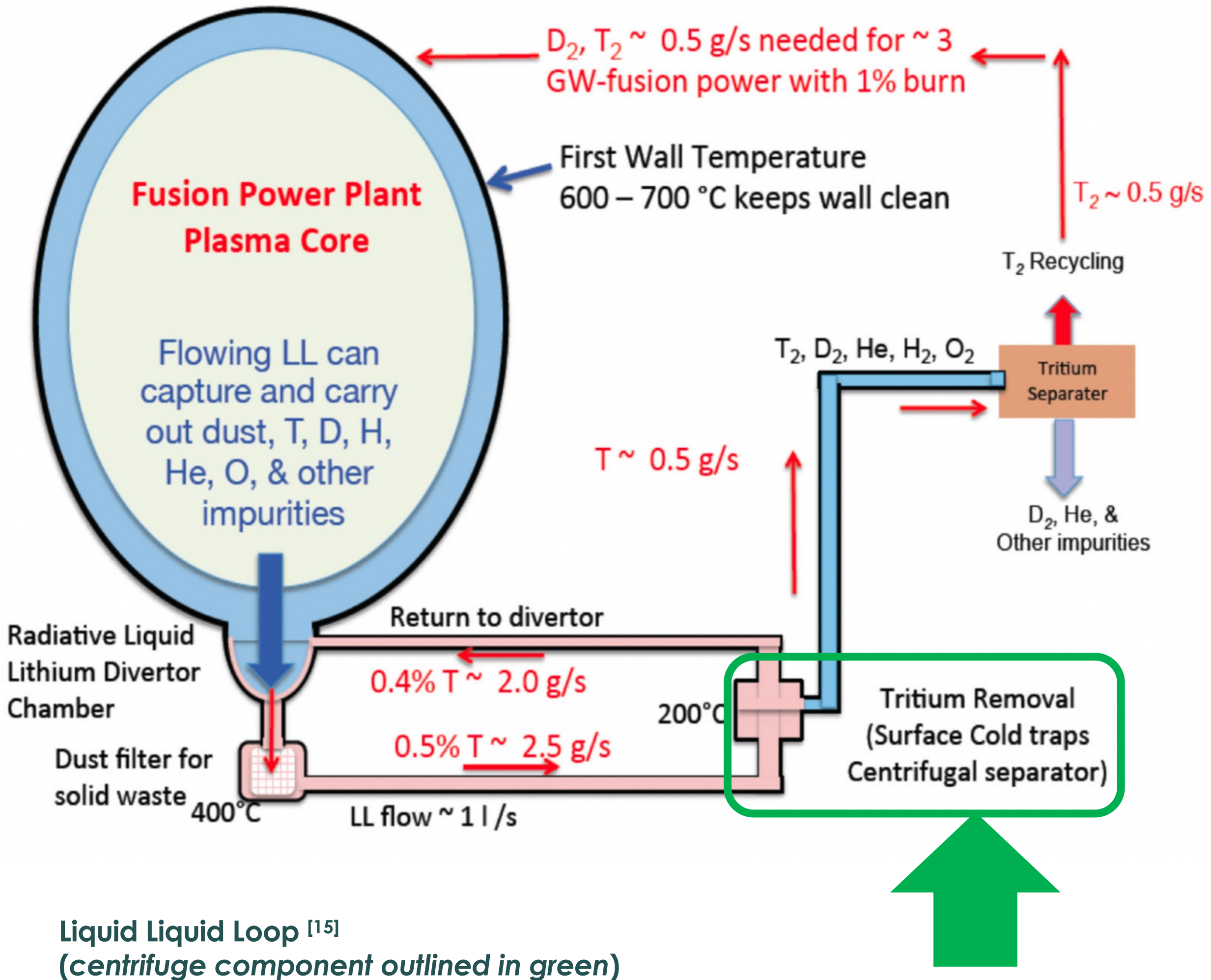
Magnetic Centrifuge

brianwynne@princeton.edu

Background (Hydride Separation)

Tritium Separation Loop

- **Liquid lithium divertor → low recycling/ high hydrogen retention [15]**
- **Pumping incident hydrogen ions from plasma with liquid lithium (Li) [3]**



Liquid Liquid Loop [15]
(centrifuge component outlined in green)

Background (Hydride Separation)

In situ concentration

- Separation and return lithium while still inside divertor region. Reduced tritium stream out → MHD drag and pumping power reduced
- Precipitate Lithium Hydride (LiH) under solubility limit (*ex. ~0.3% at 300°C → 0.044% at 200°C*)
- **Concentrate with centrifuge using density difference between LiH and Li (1.5-2X) [14]**

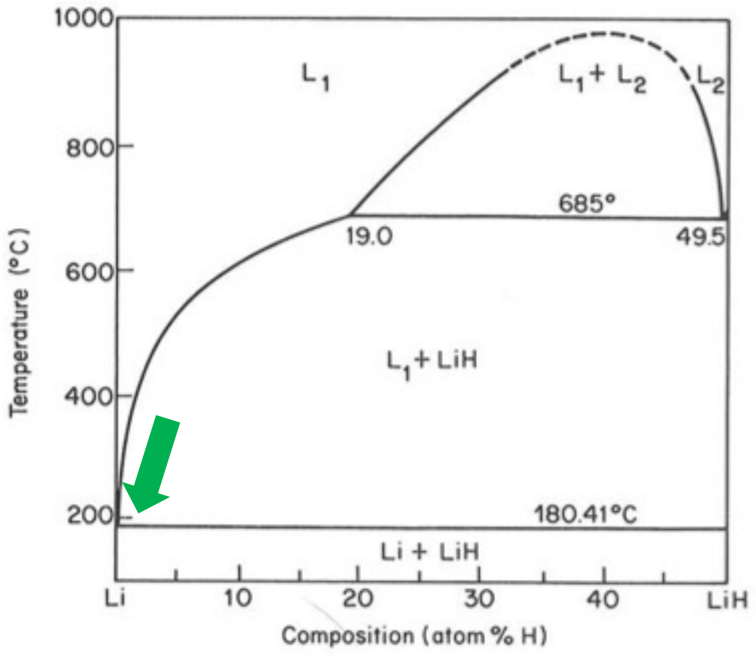
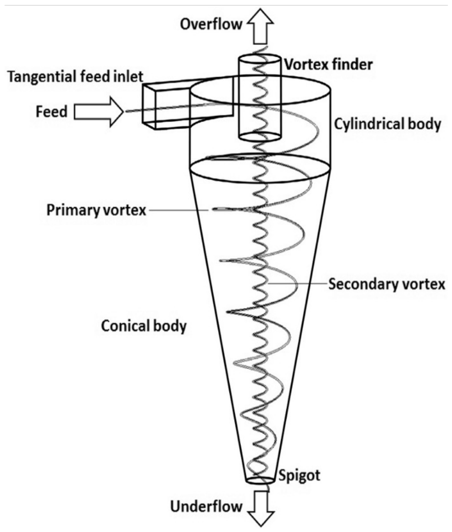


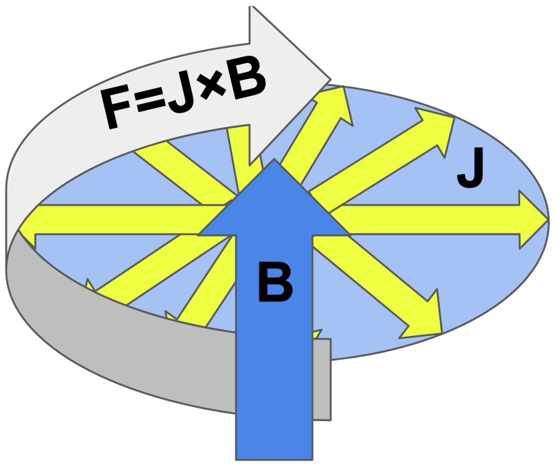
Figure 5.3 Lithium–lithium hydride phase diagram

Background Methods (Hydride Separation)

- Classic centrifuge systems
  - Spinning rotors to drive the flow rotation [11]
- Hydrocyclone
  - High tangential inlet velocity
  - Primary and secondary vortex [12]
- **Magnetic centrifuge**
  - Combined approach
  - Lorentz forces from the applied current density and magnetic field
  - J×B to drive and augment rotation



Hydrocyclone

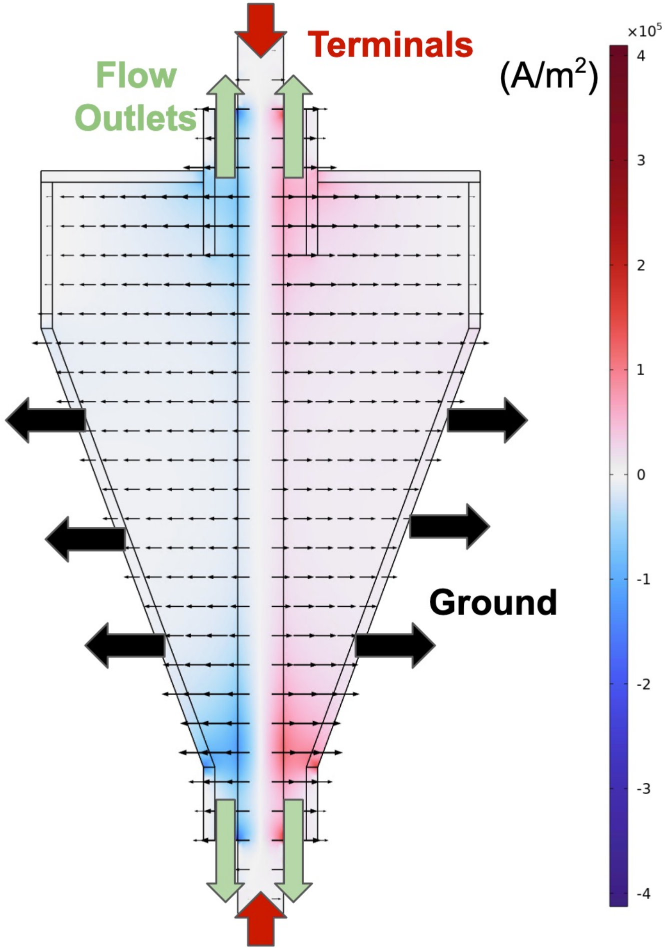


Magnetic centrifuge
Magnetic field **(B)**
Current Density **(J)**
Rotational Force **(J×B)**

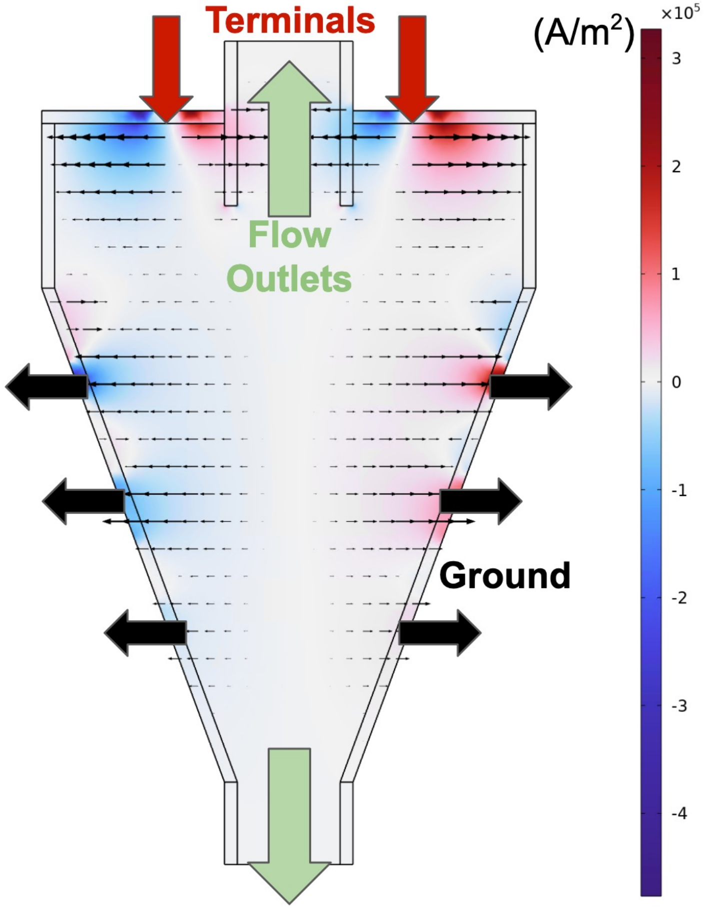
Setup (Hydride Separation)

Current density methods:

#1 Internal Rod in Center



#2 External Electrode from Above



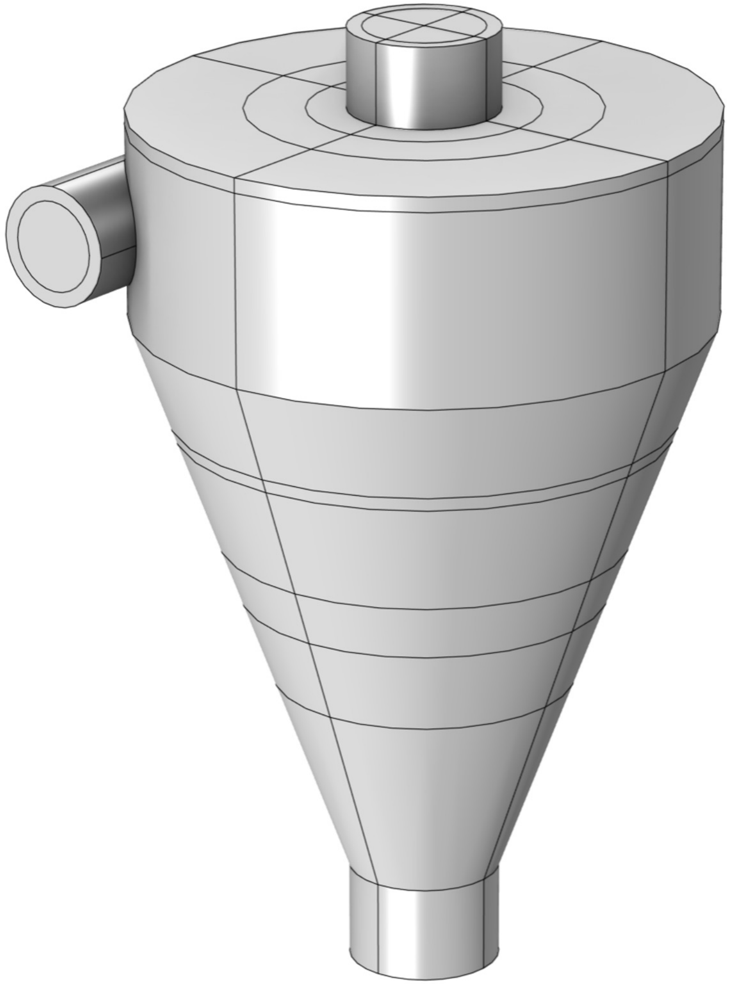
Surface: Radial Current Density

Methods (Hydride Separation)

- Magnetic Centrifuge
  - Major Radius ~3cm
  - Up to m/s velocity
  - RPM>1000
- Currently under construction

COMSOL Multiphysics

- Euler-Euler Model
  - Two-phase mixture containing a continuous (lithium) and a dispersed phase (lithium hydride)

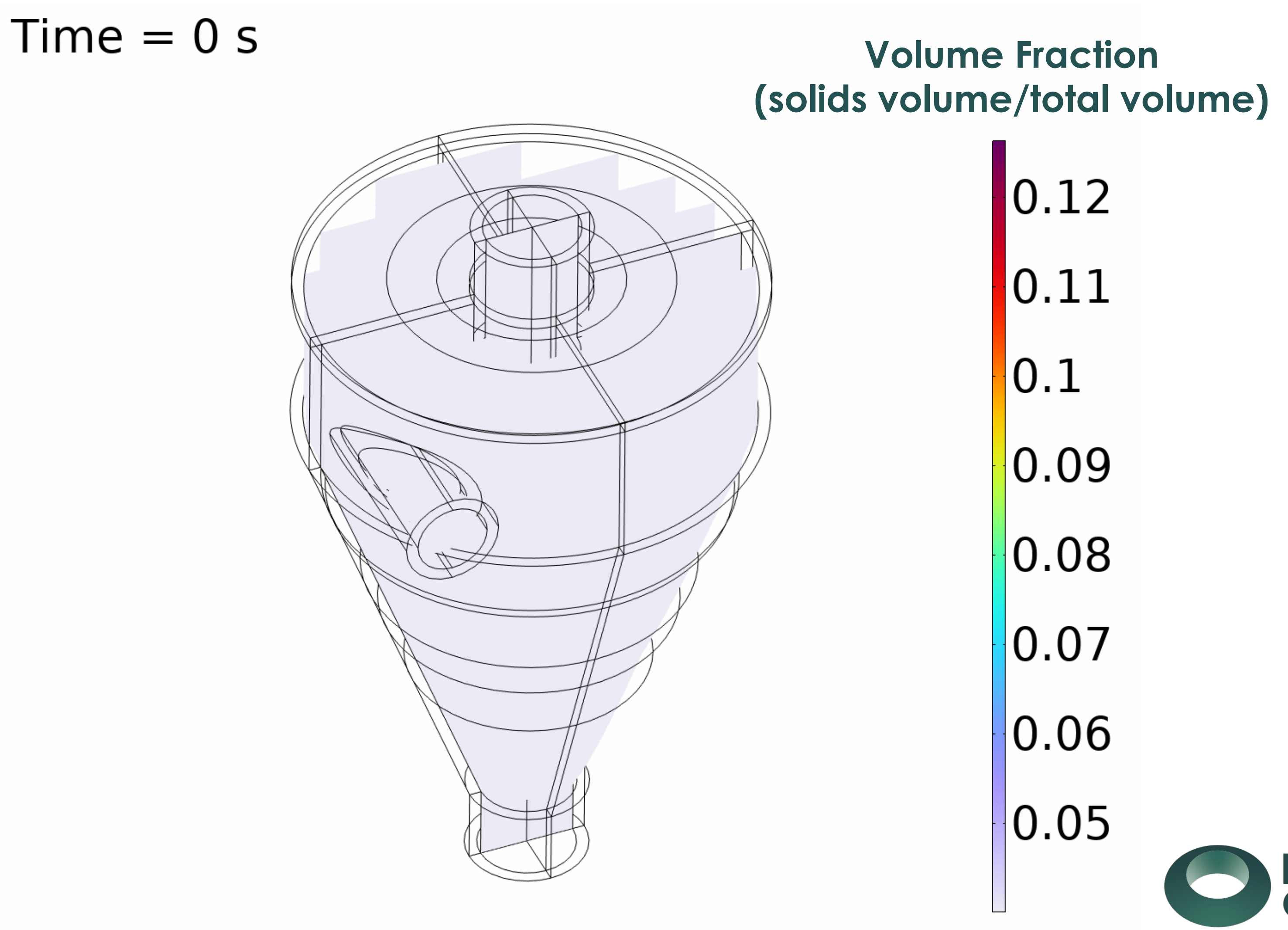


Possible Centrifuge/Hydrocyclone Design

Simulation Results (Hydride Separation)

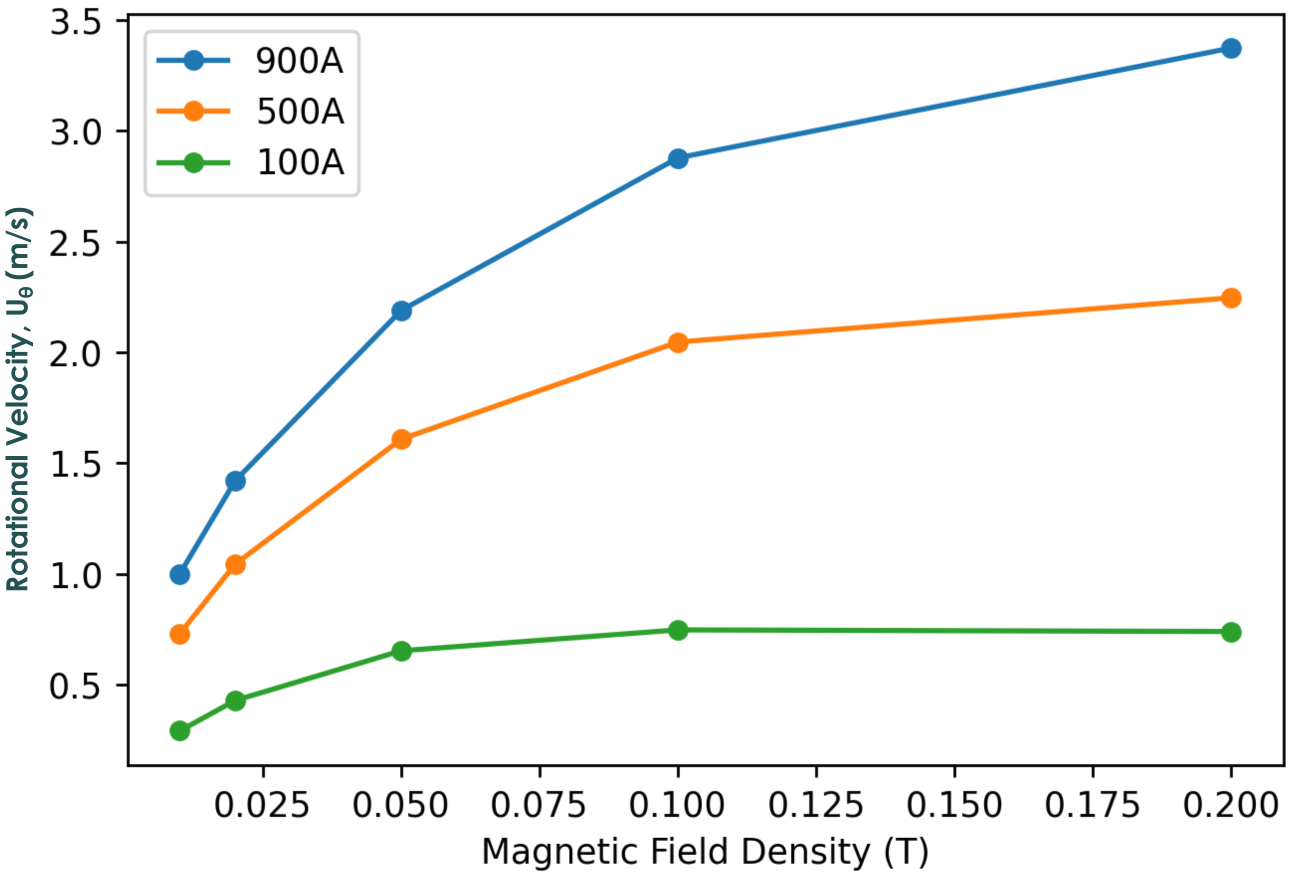
Time dependent example of separation mechanism

- Pressure and J×B Driven
  - Rotational velocity = 1.4m/s
  - Diameter = 6cm

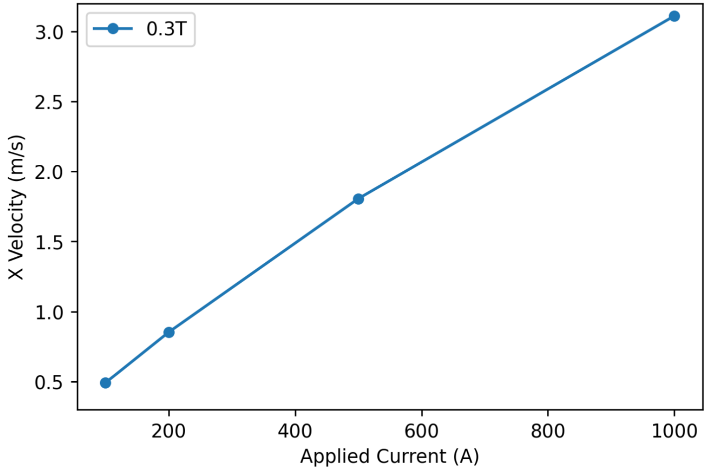


Scaling Predictions (Hydride Separation)

- Effect of Magnetic Field on Rotational Velocity



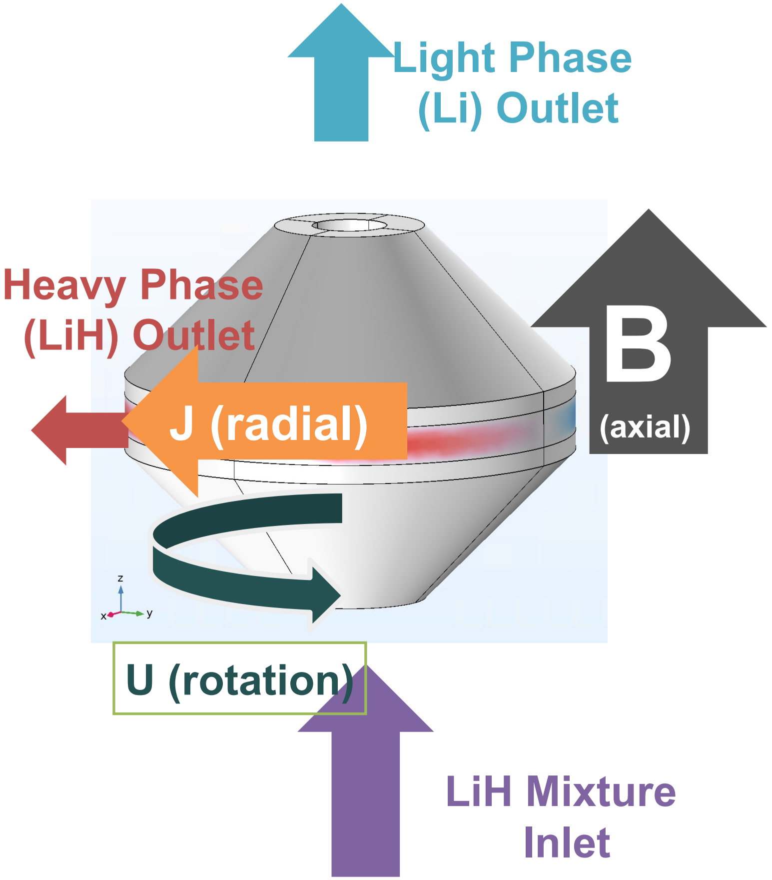
- Effect of Applied Current on Rotational Velocity



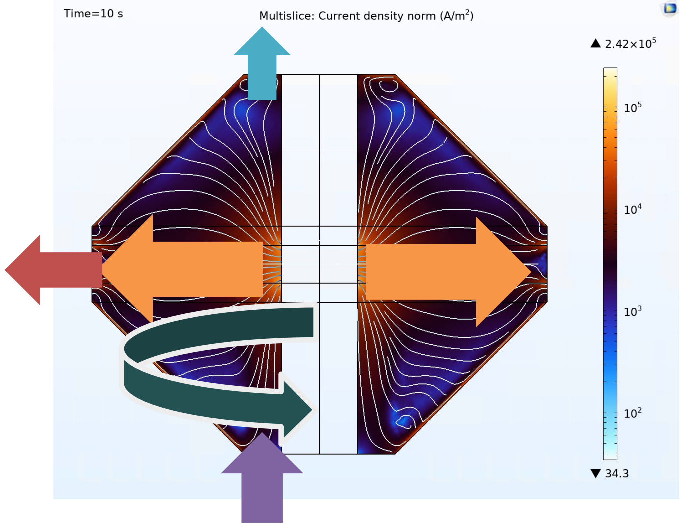
$$U_\theta \propto \sqrt{J \times B}$$

Plasma Control

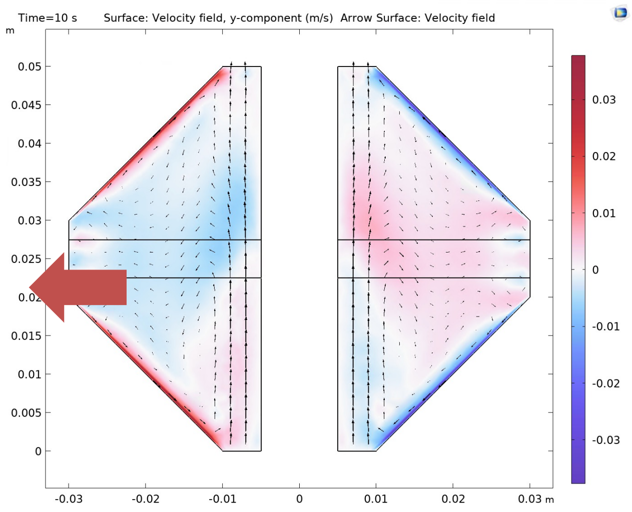
Alternative configuration (Hydride Separation)



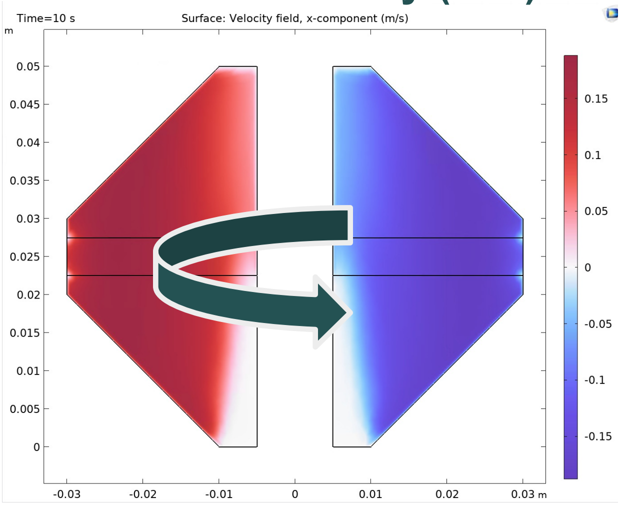
Current Density



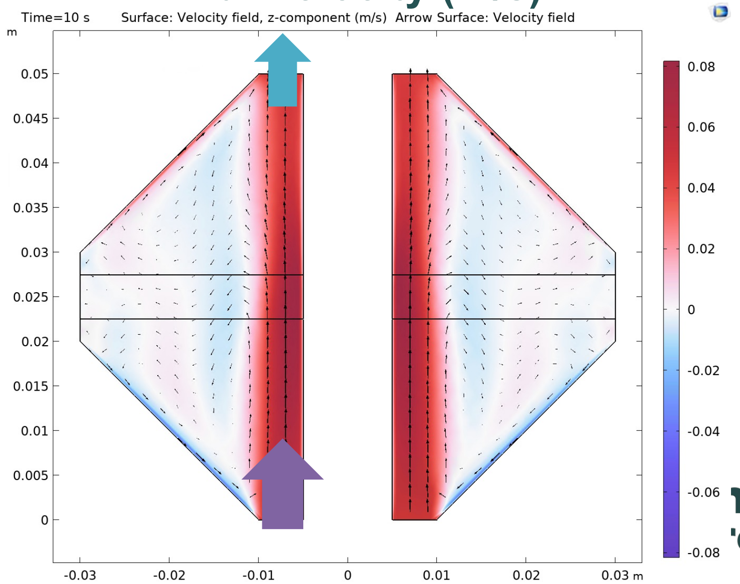
Radial Velocity (m/s)



Rotational Velocity (m/s)



Axial Velocity (m/s)



Future Work (Hydride Separation)

1. Construction of an initial design for testing at PPPL using galinstan
2. Testing the feasibility of the design to produce rotation and enable separation
3. Experimentally measure effects on:
    a. Current distribution
    b. Velocity profiles
    c. Separation efficiency
4. Testing at the University of Illinois at Urbana-Champaign with liquid lithium

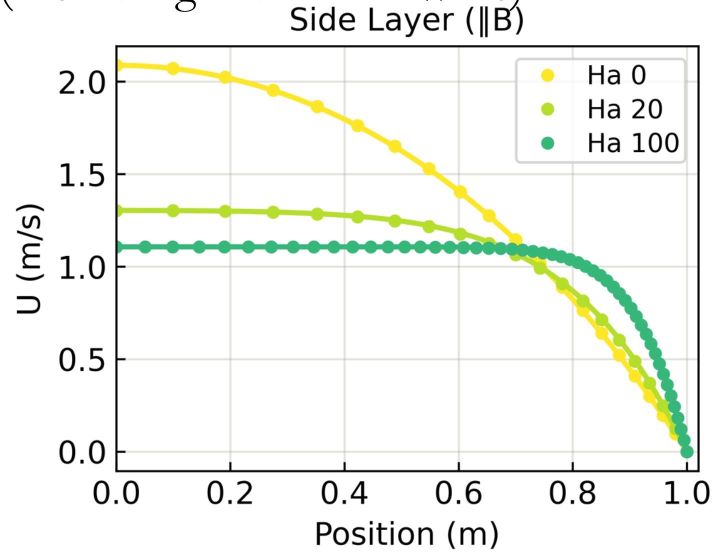
Additional Slides

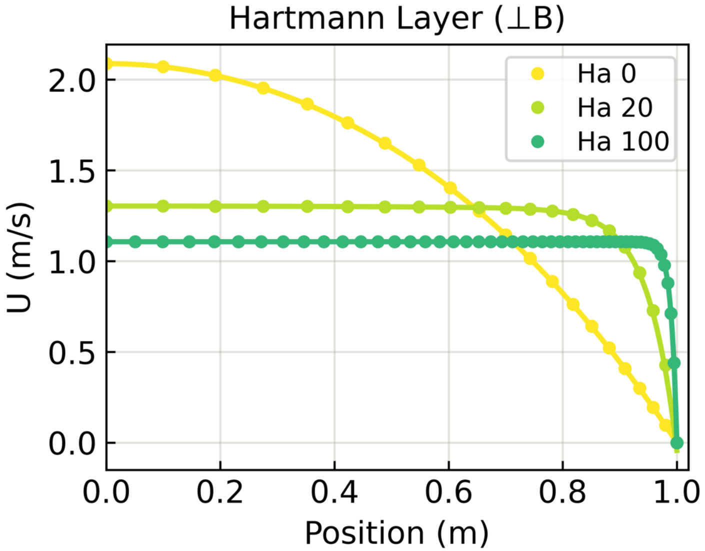
Closed Channel

brianwynne@princeton.edu

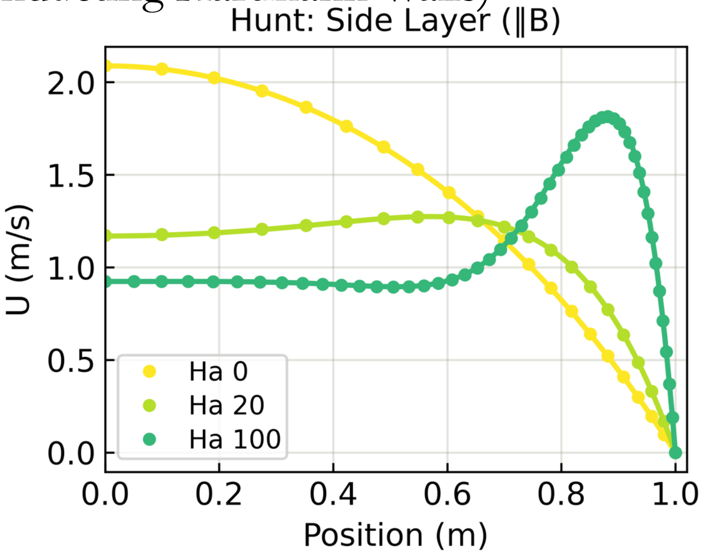
All 2D

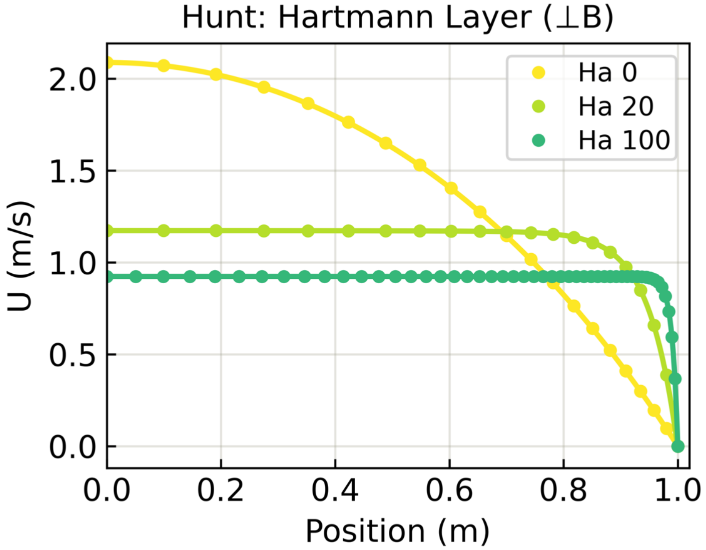
Shercliff (Insulating Hartmann Walls)





Hunt (Conducting Hartmann Walls)





Plasma Control

Closed Channel Verification

- Non-dimensional pressure drop/flow rate
  - Good way to quantify/compare to analytical solution
- Would like to get Ha=10,000
  - (currently have done up to 1,000)
- ALEX Results (Fringing B)

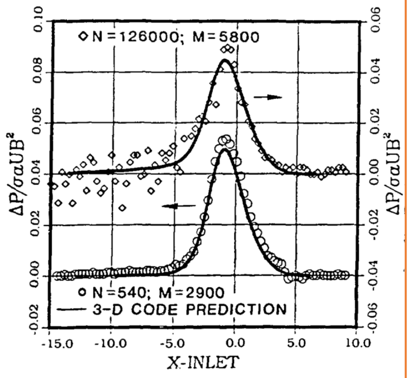


Figure 6. Transverse Pressure Difference in Rectangular Duct and Comparison with 3-D Code Prediction

Additional Slides

Extra

Turbulence (ignored for sufficient Re/Ha)

- Theoretical[70, 76] and experimental [16, 19] studies have shown that the ratio Re/Ha plays an important role in indicating the characteristics of the MHD- turbulence interactions.
- For flows bounded by electrically insulating walls, Smolentsev and Moreau [140]note that below the critical value [Re/Ha]cr = 300, flows are either laminar or exhibit quasi-two dimen- sional (Q2D) turbulence, where 3D effect are limited to regions near boundary layers.
- An experimental study of the instability of Hartmann layers by Moresco and Alboussiere showed that for conductive side walls, and insulating Hartmann walls, the critical value increases to [Re/Ha]cr = 380 [95].
- The Re/Ha ratio for flows targeted by this project falls well-below the critical value. Ha 1000, Re = rho*U*L/mu=1000*1*0.2/1=200. Re_exp=6400*0.3*0.05/2.4e-3=40000, Ha_exp = 0.3*0.05*sqrt(3.1e6/2.4e-3)=539, Re/Ha=74
  - Re_reactor = 520*10*0.05/0.5e-3=520000, Ha_reactor = 10*0.05*sqrt(3.57e6/0.5e-3)=42249, (Re/Ha)_reactor=520000/42249=12.3

Plasma Control

Parameters

- C wall parameter, Hartmann number
  - Cross section of channel and define these two

- Non-dimensional parameters
- $Re_m = UL\sigma\mu_o$
  - LMX, galinstan: (0.3)(0.05)(3.1e6)(1.257e-6)=0.085
  - Reactor, Li: (10)(0.05)(3.35e6)(1.257e-6)=2.1

- Hartmann (EM/visc), Reynolds(iner/visc), ReM, N(Ha^2/Re,EM/visc), Froude(iner/grav), also the Re_Crit=Ha/Re?? For transition to turbulence
- Hartmann,tor = Bt*(1/2*width)*sqrt(sigma/mu)
- Hartmann,normal = Bn*(h/2)*sqrt(sigma/mu)
- Froude number, Fr = U^2/(gh)
- Aspect ratio, (h/2)/(width/2)
- Prandtl number, Pr = mu Cp / k

Plasma Control

Detailed: Simplified MHD Equations

Non-dimen version? Make as simple as possible

Momentum Equation

Lorentz Force: causes MHD Drag

$$\rho(\partial\vec{u}/\partial t + (\vec{u}\cdot\nabla)\vec{u}) = -\nabla p + \mu\nabla^2\vec{u} + \rho\vec{g} + \vec{j}\times\vec{B} + \sigma_{ST}\kappa\nabla\alpha$$

Surface tension $\kappa = \nabla\cdot\left[\frac{\nabla\alpha}{|\nabla\alpha|}\right]$

Continuity of mass

$$\nabla\cdot\vec{u} = 0$$

The surface tension force acting on the interface between the two phases is modeled in this work using the continuum surface force (CSF) method [18]. Using this model, the surface tension force is given by

$$F_{st} = \sigma_{st}\kappa\nabla\alpha$$

Ohm's Law

Induced Current

$$\vec{j} = \sigma(\vec{E} + \vec{u}\times\vec{B})$$

How actually solves this, issues with gradient/derivatives

Conservation of charge

$$\nabla\cdot\vec{j} = 0$$

Resistive MHD Assumpti

- Finite electrical resistivit
- Incompressible
- Inductionless [1]

26 | MATHEMATICAL MODEL

where the curvature on the interface between the two fluids is given by

[1] U. Muller and L. Buhler, Magnetofluiddynamics in Channels and Containers, 1st ed. Berlin, Germany: Springer, 2013.

Surface Tension Effect

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mathbf{F} + \mu\nabla^2\mathbf{u}$$

$$\nabla\cdot\mathbf{u} = 0.$$

$$T = -p\mathbf{I} + \mu[\nabla\mathbf{u} + (\nabla\mathbf{u})^T] = -p\mathbf{I} + 2\mu\mathbf{E}$$

$$\mathbf{E} = 1/2\ [\nabla\mathbf{u} + (\nabla\mathbf{u})^T]$$

The normal stress balance at a free surface must be balanced by the curvature force associated with the surface tension:

$$\mathbf{n}\cdot\mathbf{T}\cdot\mathbf{n} = \sigma\ (\nabla\cdot\mathbf{n})$$

$$\mathbf{n}\cdot\mathbf{T}\cdot\mathbf{t} = \nabla\sigma\cdot\mathbf{t}$$

The tangential stress at a free surface must balance the local surface tension gradient

Plasma Control

Induction Equation, Magnetic Reynolds number

$$\mathrm{R_m} = \frac{UL}{\eta} \sim \frac{\text{induction}}{\text{diffusion}}$$

- U is a typical velocity scale of the flow
- L is a typical length scale of the flow,
- η is the magnetic diffusivity.

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

- **B** is the magnetic field,
- **u** is the fluid velocity,

$$\nabla \times (\mathbf{u} \times \mathbf{B}) \sim \frac{UB}{L} \qquad \eta \nabla^2 \mathbf{B} \sim \frac{\eta B}{L^2}$$

$$\mathrm{R_m} = \frac{UL}{\eta}$$

$R_m << 1$, Advection is relatively unimportant, and so the magnetic field will tend to relax towards a purely diffusive state, determined by the boundary conditions rather than the flow

Induction Equation, Magnetic Reynolds number

Solver details:
Using this approach, the equations are solved separately in their respective domains, and information is exchanged across domains iteratively by imposing appropriate boundary conditions

- Slide for different section of results?
- Theoretical for closed channel
- Theoretical for non-MHD free surface

- How modeling free surface:
- Question:
  - balance of forces at free surface? MHD forces at surface?

Plasma Control